

Conference: Iterated Function Systems (IFS), Fractals, Invariant Measures and  
Applications

June 10 – 12, 2016

Dalhousie University, Halifax, Nova Scotia, Canada

Welcome to Halifax!

We are very excited to have you join this conference. There is a wide range of interesting talks, and we have also scheduled time to have open discussions about future work and collaborations. There will be participants from all levels, from undergraduate students to well-known researchers. If you are new to Halifax, there are some recommendations included and please feel free to ask the organizers for suggestions. We wish you a very fruitful conference and a pleasant stay in Halifax.

We wish to thank the Atlantic Association for Research in the Mathematical Sciences (AARMS) for funding.

Sincerely,

**Organizers:** Shafiqul Islam (UPEI), Franklin Mendivil (Acadia), Dorette Pronk (Dalhousie), and Tara Taylor (StFX).



### **Welcome on Friday night**

On Friday evening, we will be setup in the Peter Wilson Common Room in the New Academic Building (NAB) near Alex Hall (see map on last page) with some refreshments and registration packets, for those who are nearby and available.

### **Snack breaks**

Saturday and Sunday morning we will have coffee/tea and muffins/fruit starting at around 8:30. This food will stay in the room throughout the morning, but you might not get your first choice if you wait!!

We will also have some similar snacks available on Saturday afternoon during the “open discussion” period from 3:00 - 3:45 pm.

### **Dinner on Saturday night**

Dinner on Saturday evening is at the Wooden Monkey ([www.thewoodenmonkey.ca/](http://www.thewoodenmonkey.ca/)), a restaurant that includes a wide variety of local and organic offerings. There are two locations, we are going to the one that is in the Alderney Ferry Terminal in Dartmouth so that we can enjoy the ferry ride across the Halifax Harbour. The reservation is for 7:30. We can take the 7:15 ferry across (\$2.50 each way). Some of us will walk down from campus, but you can meet us at the ferry terminal. The dinner is optional and we can choose any items from the menu. Spouses are welcome.

All talks are in the Colloquium room (Room 319) of the Chase building on the Dalhousie campus. See the map on the last page of this program for the location of the Chase building.

### **Wireless internet on Dalhousie campus**

Dalhousie University participates in *eduroam* (<https://wireless.dal.ca/eduroam.php>).

## Complete Schedule

### Friday, June 10, 2016

Time	Program	Location
7:00 pm – 8:30 pm	Welcome and Registration	Alex Hall, Kings College residence

### Saturday, June 11, 2016

Time	Program/talk/discussion/break	Location
8:30 am – 9:00 am	Registration	Chase Build. (colloquium room)
9:00 am – 10:00 am	M. F. Barnsley (ANU, Australia))	Chase Build. (colloquium room)
10:00 am – 10:45 am	Ö. Stenflo (Uppsala)	Chase Build. (colloquium room)
10:45 am – 11:30 am	I. Garcia (Waterloo)	Chase Build. (colloquium room)
11:30 am – 1:30 pm	<i>Lunch break</i>	
1:30 pm – 2:15 pm	F. Mendivil (Acadia)	Chase Build. (colloquium room)
2:15 pm – 3:00 pm	K. Hare (Waterloo)	Chase Build. (colloquium room)
3:00 pm – 3:45 pm	Break & open discussion	Chase Build. (colloquium room)
3:45 pm – 4:30 pm	T. Hurth (Toronto)	Chase Build. (colloquium room)
4:30 pm – 5:00 pm	R. Dawson (St. Mary's)	Chase Build. (colloquium room)
5:00 pm – 7:30 pm	free time	
7:30 pm – 9:00 pm	Dinner at Wooden Monkey	Take the 7:15 ferry

### Sunday, June 12, 2016

Time	Program/talk/discussion/break	Location
9:00 am – 10:00 am	R. L. Devaney (Boston)	Chase Build. (colloquium room)
10:00 am – 10:45 am	P. Gora (Concordia)	Chase Build. (colloquium room)
10:45 am – 11:30 am	M. Almutairi (Acadia)	Chase Build. (colloquium room)
11:30 am – 12:15 pm	E. Curry (Acadia)	Chase Build. (colloquium room)
12:15 pm – 12:30 pm	Closing remark by organizers	Chase Build. (colloquium room)

## Abstracts

### 1. IFS Theory inside out and inverse limits

Michael F. Barnsley, Australian national University, Australia.

**Abstract.** I will discuss some recent work, joint with Andrew Vince, that at first sight concerns the following question. When does there exist a unique contractive IFS  $F$ , comprising a finite set of contractive similitudes on  $\mathbb{R}^2$  such that if  $G$  is any such IFS with the same attractor, with nonempty interior, and both  $F$  and  $G$  obey the OSC, then for each  $g \in G$  there are  $f_{i_1}, f_{i_2}, \dots, f_{i_K} \in F$  such that  $g = f_{i_1} \circ f_{i_2} \circ \dots \circ f_{i_K}$ ? The presentation will be made in the context of the IFS illustrated in Figure 1.

It emerges that this work concerns certain inverse limits and the uniqueness of associated tilings.



Figure 1:

### 2. V-variable image compression

Örjan Stenflo, Uppsala University, Sweden

**Abstract:** V-variable fractals, where V is a positive integer, are intuitively fractals with at most V different “forms” or “shapes” at all levels of magnification. In this talk we describe how the concept of V-variability can be used for the purpose of image compression.

This is a joint work with Franklin Mendivil.

### 3. Assouad dimensions of complementary sets.

Ignacio Garcia, Universidad Nacional de Mar del Plata, Argentina and Waterloo

**Abstract.** Given a positive, decreasing sequence  $a$ , whose sum is  $L$ , we consider all the closed subsets of  $[0, L]$  such that the lengths of their complementary open intervals are in one to one correspondence with the sequence  $a$ . The sets in this class have zero Lebesgue measure. In this talk I'm going to discuss the possible values that Assouad-type dimensions can attain for this class of sets. In many cases, the set of attainable values is a closed interval whose endpoints we determine. This is a joint work with Kathyn Hare and Franklin Mendivil.

### 4. Multifractal and “local” zeta functions of fractals

Franklin Mendivil, Acadia University, Nova Scotia, Canada

**Abstract:** Lapidus developed his theory of fractal strings and their associated zeta functions while investigating the harmonics of regions with fractal boundary. The poles of the geometric zeta function are the “complex dimensions” of the fractal and can indicate “oscillations” in the geometry.

In this talk, we discuss two extensions of his work. The first is a zeta function for the multifractal spectrum and the second is for a “local” version of the geometric zeta function. The presentation will start with a brief review of fractal strings, zeta functions and their relations with the geometry of fractal sets.

This is joint work with J. Levy-Vehel from INRIA in France.

### 5. Local dimension of self-similar measures with overlap

Kathryn Hare, University of Waterloo, Ontario, Canada.

**Abstract.** It is well known that the set of local dimensions of a self-similar measure that satisfies the open set condition (OSC) is a closed interval. If the OSC is not satisfied, the structure of the set of local dimensions is much less well understood and can be quite different. We will show that for measures which satisfy the weaker condition, ‘finite type’, there is a subset of full measure, that is often all but the endpoints of the support of the measure, such that the set of local dimensions at points in this subset is a closed interval. As one application, we see that biased Bernoulli convolutions with contraction factor the inverse of a simple Pisot number always admit an isolated point in their set of local dimensions.

### 6. Invariant densities for randomly switched ODEs

Tobias Hurth. University of Toronto, Ontario, Canada.

**Abstract.** In this talk, we will consider some aspects of the ergodic theory for dynamical systems that arise from switching between finitely many deterministic flows at random times. In particular, we will formulate conditions under which there is a unique invariant measure that is also absolutely continuous. For a particular

system involving two linear vector fields in two dimensions, we will describe how the switching rate determines whether the density of the invariant measure is bounded. The talk is based on work with Yuri Bakhtin, Sean Lawley and Jonathan Mattingly.

7. **A fractal in the large arising from the classification of tricycloids**

Robert Dawson, Saint Mary's University, Halifax, Nova Scotia, Canada.

**Abstract:** A polycycloid is a parametric curve obtained as the sum of uniformly rotating vectors, that is, of the form  $\sum_k = 1^n R_k \exp(i(a_k t + \phi_k))$ . Where the number of summands is two, three, (etc). we call it a bicycloid, tricycloid, (etc.).

These elegant curves have popped up in various places, including toys, engineering and computer graphics. Farris (1996) gave conditions for tricycloids to have rotational symmetry. The presenter and Mr. Matthew Madill have obtained conditions for them to be mirror-symmetric, and for tricycloids with the same radii  $R_k$  and angular velocities  $a_k$ , but different phases  $\phi_k$ , to be congruent. The conditions involve rather more (elementary) number theory than one might expect, and give rise to a fractal in the large with partial self-similarity.

8. **Cantor and Sierpinski, Julia and Fatou: Crazy Fractal Topology in Complex Dynamics**

Robert L. Devaney, Boston University, Boston, USA.

**Abstract:** In this talk, we shall describe some of the rich topological structures that arise as Julia sets of certain complex functions, including the exponential and rational maps. These objects include Cantor bouquets, indecomposable continua, and Sierpinski curves. In each case, we will describe the structure of the given set and then show how it emerges in the dynamical plane for the corresponding map.

9. **Maps with memory and a two dimensional family of transformations with very diverse behaviour.**

Pawel Góra, Concordia University, Montreal, Canada

**Abstract:** I will talk about a process called map with memory and a two dimensional family of transformations induced by it. The family exhibits most diverse behaviour: from absolutely continuous invariant measure to globally attracting fixed point and singular Sinaj-Ruelle-Bowen measure. I hope to show a multitude of pictures as the study is mostly based on computer experiments.

This is a joint work with Abraham Boyarsky, Zhenyang Li and Harald Proppe

10. **Investigating the usefulness of multidimensional radix representations for computing**

Malak ALmutairi, Acadia University, Nova Scotia, Canada.

**Abstract.** This talk is an extension of typical numeration systems to the multidimensional setting. We investigate the complexity of a generalized addition algorithm

for computation with multidimensional digit representations. Positional digit representations in one dimension, such as base 10 or base 2, were a major historical advance for arithmetic. The standard representation of vectors involves multiple components, rather than being purely a digit representation, so our research asks one main question: are multidimensional digit representations computationally useful?

In this talk, we investigate the algorithmic complexity of binary additions because they are the simplest way to analyze the idea of algorithmic complexity and to demonstrate algorithms that do or do not terminate, and have linear or quadratic complexity. We also investigate the algorithmic complexity of addition involving radix representations in higher dimensions; specifically, we consider the Malak matrix as our radix in  $\mathbb{Z}^2$ . We prove our algorithm terminates, and that the time complexity is quadratic. This investigation has successfully answered the main question for just one example and it is a first step to generalize the answer for  $\mathbb{Z}^n$ .

#### 11. **Martin boundary theory for wavelet multiresolution analysis**

Eva Curry, Acadia University, Nova Scotia, Canada.

**Abstract.** Characterization theorems for both scaling functions (Hernández and Weiss) and low-pass filters (Gundy) associated with multiresolution analysis (MRA) wavelets have been given, and have been generalized to the multidimensional setting (Curry). Some of the characterization conditions provided initially appear very technical and lacking in a clear conceptual framework. These conditions may be better understood in the context of Martin boundary theory for Markov processes on  $n$ -ary trees, and we show a new characterization in this setting. In particular, we show that a low-pass filter generates an uncountable family of Markov processes on  $n$ -ary trees. Conversely, a single process satisfying a set of necessary conditions can be used to define a low-pass filter and scaling function. The MRAs so defined may be only a subset of possible MRAs; in general, an uncountable family of Markov processes on trees satisfying a gluing condition are needed to recover any low-pass filter and scaling function. We will discuss the properties of the MRAs defined using a single process, as well as the connections between our families of processes on trees and the solenoids of Dutkay and Jorgensen in their study of generalizations of multiresolution analyses.

## **List of speakers and participants**

Malak ALmutairi (Acadia University)

Michael F. Barnsley (Australian National University )

Erica Caines (Acadia University)

Eva Curry (Acadia University)

Robert Dawson (Saint Mary's University)

R. L. Devaney (Boston University)

Ewa Duma (Concordia University)

Ignacio Garcia (Universidad Nacional de Mar del Plata, Argentina and Waterloo)

Pawel Gora (Concordia University)

Kathryn Hare (University of Waterloo)

Tobias Hurth (University of Toronto)

Shafiqul Islam (University of Prince Edward Island)

Franklin Mendivil (Acadia University)

Dorette Pronk (Dalhousie University)

Sean Rowley (St. Francis Xavier University)

Orjan Stenflo (Uppsala University, Sweden)

Tara Taylor (St. Francis Xavier University)

Hayden Vanlasterstine (University of Prince Edward Island)

Tony Vargas (Dalhousie University)



## Useful information

### Information:

- Pick up a copy of the free local weekly newspaper, The Coast, or read it online at [www.thecoast.ca](http://www.thecoast.ca). It has lots of information about live music, art galleries, theater and restaurants.
- Explore Nova Scotia website: [www.novascotia.com/explore](http://www.novascotia.com/explore)

### Getting around:

Halifax is a very walkable city. Many attractions (Spring Garden Road, Citadel Hill, the waterfront, Point Pleasant Park, Halifax Seaport Farmers Market, Public Gardens) are within walking distance of the conference. Bus and ferry information can be found at [www.halifax.ca/transit/Schedules/index.php](http://www.halifax.ca/transit/Schedules/index.php). Bus route 320 goes to the airport.

### Restaurants/Pubs:

There are many wonderful restaurants within walking distance of campus. You can either head East down Spring Garden, North up to the Quinpool area, or the waterfront. Both areas have many choices in terms of ethnicity, cost, etc. Some favourites:

1. Coburg Coffee House (cafe very close to campus, free wifi): 6085 Coburg Rd, (902)429-2326, [www.coburgcoffee.com/](http://www.coburgcoffee.com/)
2. Wild Leek (vegan): 2156 Windsor St, (902)444-5466, [wildleek.ca](http://wildleek.ca)
3. Wasabi House (sushi): 6403 Quinpool Road, (902)429-3300, [wasabihouse.ca/wasabi-house-halifax/](http://wasabihouse.ca/wasabi-house-halifax/)
4. Mezza (Lebanese): 6386 Quinpool Road, (902) 444-3914, [www.mezzalebanesekitchen.com/](http://www.mezzalebanesekitchen.com/)
5. Heartwood (vegetarian/vegan): 6250 Quinpool Road, (902)425-2808, [www.iloveheartwood.ca/](http://www.iloveheartwood.ca/)
6. The Ardmore Tea Room (old-fashioned diner): 6499 Quinpool Rd, (902)423-7523, [www.facebook.com/Ardmoretearoom](http://www.facebook.com/Ardmoretearoom)
7. Stillwell (Craft beer and snacks): 1672 Barrington St, (902) 421-1672, [www.barstillwell.com/](http://www.barstillwell.com/)
8. Gahan House (Craft beer and quality pub food): 1869 Upper Water St, (902)444-3060, [halifax.gahan.ca/](http://halifax.gahan.ca/)

9. McKelvies (seafood): 1680 Lower Water St, (902)421-6161, [mckelvies.com](http://mckelvies.com)
10. Murphy's (seafood): 1751 Lower Water St, (902)420-1015, [www.mtcw.ca/](http://www.mtcw.ca/)
11. Salty's (seafood): 1877 Upper Water St, (902)423-6818, [saltys.ca/](http://saltys.ca/)
12. Pete's Frootique (great to get fresh healthy food to go): 1515 Dresden Row, (902)425-5700, [petes.ca/](http://petes.ca/)
13. Economy Shoe Shop (unique bar/restaurant): 1663 Argyle St, (902)423-8845, [www.economyshoeshop.ca/](http://www.economyshoeshop.ca/)
14. Henry House (local craft beer and quality pub food): 1222 Barrington St, (902)423-5660, [www.henryhouse.ca/](http://www.henryhouse.ca/)
15. The Stubborn Goat (gastropub): 1579 Grafton St, (902)405-4554, [www.stubborngoat.ca/halifax/](http://www.stubborngoat.ca/halifax/)
16. The Bicycle Thief (quality Italian food): 1475 Lower Water St, (902)425-7993, [bicyclethief.ca/](http://bicyclethief.ca/)
17. The Old Triangle(Irish pub with food and live music): 5136 Prince St, (902)492-4900, [www.oldtriangle.com/welcome/](http://www.oldtriangle.com/welcome/)

**Attractions:**

1. Point Pleasant Park: good for walking, running, a natural escape from the city [www.novascotia.com/see-do/attractions/point-pleasant-park/1461](http://www.novascotia.com/see-do/attractions/point-pleasant-park/1461)
2. Public Gardens [www.halifax.ca/publicgardens/](http://www.halifax.ca/publicgardens/)
3. Halifax Seaport Farmers' Market (on the waterfront): [www.halifaxfarmersmarket.com/](http://www.halifaxfarmersmarket.com/)

NAB

<https://www.google.com/maps/embed?pb=!1m2!1m8!1m3!1d2839.053622567448!2d-63.5918...>



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