Assignment #4

Date Due: April 4, 2017

Total: 100 marks

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1. (10 marks) Write a grammar that generates the language

$$\{w \in \{a,b\}^* \mid |w|_a = |w|_b - 1\}.$$

2. (10 marks) Prove that the following grammar is ambiguous $S \to A{*}B|a\%A\%a,\ A \to A{*}A|aa,$ $B \to bb.$

3. (10 marks) Write an equivalent grammar for the following DFA

```
(START) |- 0
0 a 1
0 b 2
1 a 0
1 b 6
2 b 3
2 a 5
3 a 4
3 b 6
4 a 3
4 b 2
5 a 2
5 b 4
6 b 5
6 a 7
7 a 7
7 b 7
1 -| (FINAL)
4 -| (FINAL)
6 -| (FINAL)
```

4. (10 marks) Construct an equivalent DFA for the following grammar

```
S->aA
S->bB
S->bS
S->aC
A->b
A->c
A->a
A->aB
B->b
B->bC
C->bA
C->b
C->a
C->aB
C->aC
```

5. (20 marks maximum) Prove that the following languages are context free:
   (a) (10 marks) $\{a^{n+1}b^{m+2}c^{n+4} \mid m, n \geq 0\}$
   (b) (10 marks) $\{a^{n+2}b^{n}c^{m}d^{m+2} \mid m, n \geq 0\}$
   (c) (10 marks) $\{uc^{n}v \mid |v|_a + 3|v|_b = 3|u|_a + |u|_b, n \geq 1\}$
6. (20 marks) Given the following grammar:

```
S -> aS | bS |aAaA |BbAb
A -> aB | bC | a
B -> aA | bC |AB |a
C -> aA | bB | aa  | b
```

   (a) Construct the PDA that accepts the same language by empty stack.
   (b) Construct an equivalent PDA that accepts the same language by final states.
7. (maxim 25 marks) Prove that the following languages are not context free:
   (a) (10 marks) $\{a^{p-3} \mid p \text{ is prime}, p > 5\}$
   (b) (10 marks) $\{a^{4n}b^{3n}c^{2n} \mid n \geq 4\}$
   (c) (10 marks) $\{a^{n^4+2n^2} \mid n \geq 2\}$
8. (maxim 20 marks) Are the following language context free?
   (a) (15 marks)
   $L = \{w \in \{a,b\}^* \mid w = a^{n^2}b^n, n \geq 0 \text{ and } |w|_a \equiv 3 \mod 5, |w|_b \equiv 1 \mod 6\}$
   (b) (15 marks)
   $L = \{w \in \{a,b\}^* \mid w = b^n a^n b^m, n > m \geq 0\}\{w \in \{a,b\}^* \mid w = b^m a^n b^n, m, n \geq 0\}$

   The proof must be correct to receive points.