Assignment #1

Date Due: February 7, 2018

Total: 100 marks

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1. (20 marks) Give a DFA accepting the following language over the alphabet $\{0, 1\}$:
   (a) the set of all strings beginning with 010 and ending in 10110.
   (b) the set of all strings beginning with 1010 and ending in 0101.

2. (40 marks) Give DFAs accepting the following languages over the alphabet $\{0, 1\}$:
   (a) the set of all strings consisting of alternating groups of 11 and 10 (11 and 10 *alternates*);
   (b) the set of all strings whose fifth symbol from the right end is a 1;
   (c) the set of strings that either begin, or end (or both) with 1110;
   (d) the set of strings such that the number of 0's is divisible by five, and the number of 1's is divisible by six.

3. (20 marks) Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 1, 2, 3, 7\}$:
   (a) the set of all strings beginning with a 1, 2 or 3, that, when the string is interpreted as an integer in base 9, is a multiple of 6 plus 4. For example:
      - strings 11, 31,37 ,1111, 3001, ~~707~~, 301, 3331, 22211 and 22277 are in the language;
      - the strings 10, 00,011, 0010, 36 , 13, 23, 113, 1313, 2347,2, 21, 161, 3333, 707, and 041 are not.

   (b) The set of all strings that ends with an 1, 2, or 3 and when the string is interpreted ***in reverse*** as an integer in base 9, is a multiple of 6 plus 4.

      Examples of strings in the language are 11, 13,73 ,1111, 1003, ~~707~~, 103, 1333, 11222 and 77222

      Examples of strings that are not in the language are: 0, 00, 01, 110, 0100,63, 31, 32, 161,311, 3131,7432, 2, 12, 3333, 707, and 140 .

4. (20 marks) Let $p \in \mathbb{N}$, p prime, $p > 4$, and $h \in \mathbb{N}$ such that $1 \le h < p$. We have a DFA $A = (\Sigma, Q, \delta, 0, F)$ with $Q = \{0, 1, \ldots, k\}$, $k \ge p$, $\{a, b\} \subseteq \Sigma$. We have that $\delta(q, a) = q + h \bmod p$, for all states $q \in Q$. In these conditions:
   (a) show by induction on n that for all $n \ge 0$ and $q < p$, $\overline{\delta}(q, a^{n \cdot p}) = q$;

(b) show that either $\{a^p\}^*baa^pab \subseteq L(A)$, or $\{a^p\}^*baa^pab \cap L(A) = \emptyset$.

5. (10 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow$ 0	1	0
$*$ 1	2	1
2	3	2
3	0	3

Informally describe the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

6. (10 marks) Repeat the above exercise for the following transition table:

	0	1
$\rightarrow$ A	B	A
B	C	B
C	D	C
$*$ D	B	D

The maximum is bounded to 115 marks.

Very Important: Verify your solutions using Grail; describe ***how do you think*** for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to ***all*** exercises in class.