

Assignment #4

Date Due: April 4, 2017

Total: 100 marks

1. (10 marks) Write a grammar that generates the language

$$\{w \in \{a, b\}^* \mid |w|_a = |w|_b - 1\}.$$

2. (10 marks) Prove that the following grammar is ambiguous $S \rightarrow A^*B|a^*A^*a$, $A \rightarrow A^*A|aa$, $B \rightarrow bb$.

3. (10 marks) Write an equivalent grammar for the following DFA

(START) - 0	4 a 3
0 a 1	4 b 2
0 b 2	5 a 2
1 a 0	5 b 4
1 b 6	6 b 5
2 b 3	6 a 7
2 a 5	7 a 7
3 a 4	7 b 7
3 b 6	1 - (FINAL)
	4 - (FINAL)
	6 - (FINAL)

4. (10 marks) Construct an equivalent DFA for the following grammar

S → aA	A → aB
S → bB	B → b
S → bS	B → bC
S → aC	C → bA
A → b	C → b
A → c	C → a
A → a	C → aB
	C → aC

5. (20 marks maximum) Prove that the following languages are context free:

- (a) (10 marks) $\{a^{n+1}b^{m+2}c^{n+4} \mid m, n \geq 0\}$
- (b) (10 marks) $\{a^{n+2}b^n c^m d^{m+2} \mid m, n \geq 0\}$
- (c) (10 marks) $\{uc^nv \mid |v|_a + 3|v|_b = 3|u|_a + |u|_b, n \geq 1\}$

6. (20 marks) Given the following grammar:

S \rightarrow aS | bS | aAaA | BbAb
 A \rightarrow aB | bC | a
 B \rightarrow aA | bC | AB | a
 C \rightarrow aA | bB | aa | b

- (a) Construct the PDA that accepts the same language by empty stack.
- (b) Construct an equivalent PDA that accepts the same language by final states.

7. (maxim 25 marks) Prove that the following languages are not context free:

- (a) (10 marks) $\{a^{p-3} \mid p \text{ is prime}, p > 5\}$
- (b) (10 marks) $\{a^{4n}b^{3n}c^{2n} \mid n \geq 4\}$
- (c) (10 marks) $\{a^{n^4+2n^2} \mid n \geq 2\}$

8. (maxim 20 marks) Are the following language context free?

- (a) (15 marks)
 $L = \{w \in \{a, b\}^* \mid w = a^{n^2}b^n, n \geq 0 \text{ and } |w|_a \equiv 3 \pmod{5}, |w|_b \equiv 1 \pmod{6}\}$
- (b) (15 marks)
 $L = \{w \in \{a, b\}^* \mid w = b^n a^n b^m, n > m \geq 0\} \cup \{w \in \{a, b\}^* \mid w = b^m a^n b^n, m, n \geq 0\}$

The proof must be correct to receive points.