Assignment #3

Date Due: November 28, 2019

Total: 100 marks

For each of the algorithms designed you have to

- compute its time complexity,
- prove that the alleged time complexity corresponds to that of your algorithm,
- prove that your algorithm is optimal or at least give an argument to support your design.
- You should specify for each algorithm designed the programming technique used (brute force approach should also be mentioned). Missing the name of the programming technique may have as a result in several marks deducted.

In case you design a sub-optimal algorithm you may loose marks, depending on how slow is your algorithm, compared to an optimal one.

1. (15 marks) Write an algorithm that receives as input two n-element arrays A and B of real strict positive numbers and a value v. The algorithm returns 1 if there are i and j such that $v = A[i] * B[j]$ and 0 otherwise.
2. (10 marks) Write an algorithm for the union of two n-element sets A and B of real numbers. Sets are represented by sorted arrays.
3. (10 marks) Repeat exercise 2 for intersection.

   Write the algorithm and prove its efficiency.
4. (maximum 35 marks) Can we use BFS strategy to design a single-source shortest-path algorithm?

   Discussion.

   (a) (10 marks) Explain your algorithm in case you can do it or explain why not, by giving. strong arguments against this design including a counter example.

   (b) (10 marks) What about DFS?

   (c) (20 marks) Redo the exercise for the minimal spanning tree.

5. (30 marks) When registering for a CS course each student will know who the professor is. However, it is assumed that the professor does not have the class list and would not know anybody. On the first day of class the students and the professor are in the same room and each will carry a distinct numerical label. Your task is to identify the professor. The only questions you can ask each participant would be to provide a list of the known persons. Design an efficient algorithm to:

   (a) (10 marks) identify if there could be a professor in class.

   (b) (10 marks) identify if there is a student not registered in that class.

   (c) (15 marks) identify the professor.

6. (10 marks) Two thieves are carrying n gold coins in two bins, in a stolen car. The physical characteristics of the the gold coins are the same, but while the coins in the first bin are all the same, the ones in the second bin are different from the ones in the first bin. At one corner, the driver suddenly brakes and the coins are all mixed up. They have a device that can be applied to two coins and tells whether they are different or not. It is known in advance that most of the coins (more then 50%) are from the first bin. Find the optimal algorithm that the two thieves should apply in order tor put the coins back into the bins as fast as possible, to hide the first bin before the police will arrive. How many comparisons are necessary, in the worst case, to find at least one coin from the first bin?

   (Beware: it is possible that two coins are identical, but do not belong to the first bin.)