Assignment #1

Date Due: October 10, 2019

Total: 100 marks

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1. (20 marks) Prove that

$$(n+19)^{2019} + 19n^{2000} - 2019n^{19} = \Theta(n^{2019}).$$

by

(a) using the definition of Θ.

(b) using the limit rule.

2. (10 marks) Prove that for any real constants a, b, c, with $a, b, c > 0$, we have

$$(bn+a)^b + (a+c)n^c = \Theta(n^{\max\{b,c\}}).$$

3. (20 marks) Define a recurrence $T(n)$ for the following code

```
int f(int m, int n)
{
 if (n<=1)
    return m+2019;
    else
    return n+f(m+19,n-3);
}
```

(a) Solve the recurrence and represent it in O notation.

(b) Solve the recurrence and represent it in O notation for the case when we replace the return line by

```
return 2n+f(0,n/3);
```

4. (10 marks) Write a code corresponding to the following running time:

$$T(n) = n^2 + 2n + T(n-19).$$

Explain what statements you count and why when you compute the formula for the time complexity.

5. (10 marks, each, maximum 50)

   For the following problems use the theory learned in class. In case you have to use Master Theorem, please use the version from the slides, not the one in the textbook. You must give all the details of your work to receive full credit. Decimal approximations of irrational or periodic numbers should not be used.

   Solve the following recurrences

   (a)
$$T(n) = \begin{cases} n+2, & \text{if } n=0, n=1, n=2, \\ & n=3, \text{ or } n=4 \\ 5T(n-1) - 10T(n-2) + 10T(n-3) & \\ -5T(n-4) + T(n-5), & \text{otherwise,} \end{cases}$$

   (b) $T(n) = 9T\left(\frac{n}{3}\right) + n\sqrt{n};$

   (c) $T(n) = 36T\left(\frac{n}{6}\right) + n^2;$

   (d) $T(n) = 14T\left(\frac{n}{5}\right) + n^2;$

   (e) $T(n) = 9T(\frac{n}{5}) + \sqrt[4]{n^7};$

   (f) $T(n) = 16T\left(\frac{n}{4}\right) + (n+3)\sqrt{n-3}.$

   (g)
$$T(n) = \begin{cases} 1, & \text{if } n = 0 \\ 3T(n-1) + n^2 + 3n + 4 + n2^{n-2}, & \text{otherwise;} \end{cases}$$