

Assignment #1

Date Due: October 17, 2025

Total: 100 marks

We have the following languages:

$L_1 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ that begin with } 0101\},$

$L_2 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ that end with } 01011\},$

$L_3 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ with } 110 \text{ being a subword}\},$

$L_4 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ with an odd number of } 0\text{'s}\},$

$L_5 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ with an even number of } 1\text{'s}\},$

$L_6 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ having the fifth symbol from the right end a } 0\},$

$L_7 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ beginning with } 2102\},$

$L_8 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ ending in } 2102\},$

$L_9 = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ with the number of } 1\text{'s multiple of } 5\},$

$L_{10} = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ with the number of } 1\text{'s multiple of } 7\},$

$L_{11} = \{\text{the set of all strings over the alphabet } \{a, b\} \text{ with the number of } a\text{'s multiple of } 6\},$

$L_{12} = \{\text{the set of all strings over the alphabet } \{a, b\} \text{ with the number of } b\text{'s multiple of } 5\},$

$L_{13} = \{\text{the set of all strings over the alphabet } \{0, 1, 2\} \text{ consisting only of alternating groups of } 20 \text{ and } 01 \text{ (20 and } 01 \text{ alternates at least once)}\},$

and the following homomorphisms

$h : \{a, b\} \longrightarrow \{0, 1, 2\}^*, h(a) = 10, h(b) = 21; \text{ and } g : \{0, 1, 2\} \longrightarrow \{a, b\}^*, g(0) = a, g(1) = ba, g(2) = \varepsilon.$

1. (maximum 25 marks) Compute the languages (5 marks each)

(a) $L_{20} = L_1 \cap L_2.$

(b) $L_{21} = 01011\Sigma^* \cap \Sigma^*1010$

(c) $L_{22} = L_{13}$

(d) $L_{23} = L_6$

(e) $L_{24} = L_7 \cap L_8$

(f) $L_{25} = L_{11} \setminus L_{12}$

(g) $L_{26} = h^{-1}(L_4)$

(h) $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$

(i) $L_{28} = g(L_1^R)$

2. (maximum 55 marks, 10 marks each) For each of the following languages give a DFA accepting it over the alphabet $\{0, 1, 2\}$ or $\{a, b, c\}$, depending on the alphabet of the language.

- (a) L_{20}
- (b) L_{21}
- (c) L_{22}
- (d) L_{23}
- (e) L_{24}
- (f) L_{25}
- (g) L_{26}
- (h) L_{28}

3. (20 marks) Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 1, 2, 4, 8\}$:

- (a) the set of all strings beginning with a 1, 2 or 4, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2. For example:
 - strings 2,18,24,41,128,244,408,481,4881 and 24181 are in the language;
 - the strings 4, 8,02,04,42,44,124,404,48,2018,2418,882 and 035 are not.
- (b) The set of all strings that ends with an **1, 2, or 4** and when the string is interpreted *in reverse* as an integer **in base 9, is a multiple of 5 plus 2**.
 - Examples of strings in the language are 2,81,42,14,821,442,804,184,1884 and 18142
 - Examples of strings that are not in the language are: 4,8,20,40,24,44,421,404,84,8102,8142,288 and 530.

4. (10 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow 0$	1	3
1	2	1
* 2	1	2
3	3	4
4	4	3

Informally describe, as simple as possible, the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

The maximum is bounded to 115 marks.

Very Important: Verify your solutions using Grail (5 marks for each of exercises 2,3, and 4); describe *how do you think* for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to *all* exercises in class.