Trigonometry

(1) Degrees + Radians (2) Right angle Trig (3) plane Trig
- Unit Circle
- Graphs

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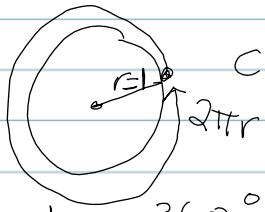
(1) Degrees & Radians

Circle: 

Circumference = $2\pi r$

$2\pi = 360^\circ \Rightarrow \pi \text{ rad} = \frac{360^\circ}{2}$

$\Rightarrow \boxed{\pi \text{ rad} = 180^\circ} \leftarrow$

Conversion: (1) ° to Rad → mult by $\frac{\pi}{180}$

(2) Rad to deg → mult by $\frac{180}{\pi}$.

eg Find degree measure of $\frac{3\pi}{4}$ rad.

$$\left(\frac{3\pi}{4}\right)\frac{180}{\pi} = \boxed{135^\circ}$$

eg Find Rad measure of 225°.

$$(225)\left(\frac{\pi}{180}\right) = \boxed{\frac{5}{4}\pi}$$

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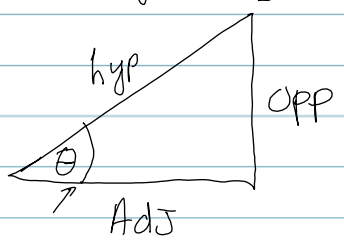
(2) Right Angle Trig

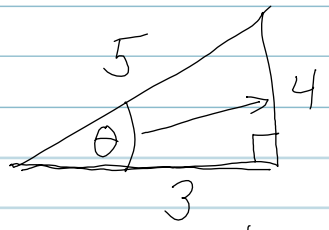


$\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, Big 3.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opp}}{\text{adj}}$.

SOH CAH TOA

$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$, $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$.

eg 

Find $\sin\theta$, $\cos\theta$, $\cot\theta$.

$\sin\theta = \frac{opp}{hyp} = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\cot\theta = \frac{adj}{opp} = \frac{3}{4}$.

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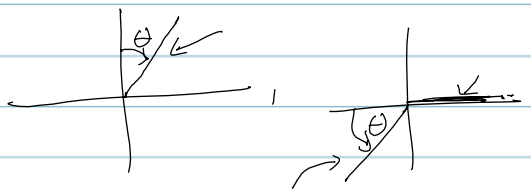
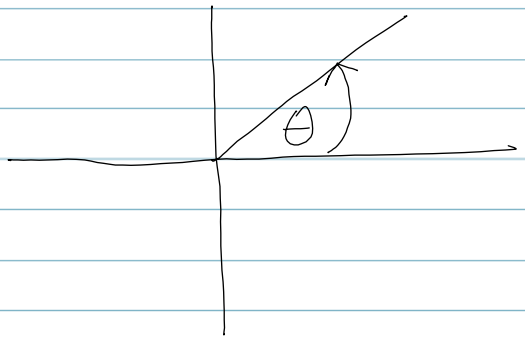
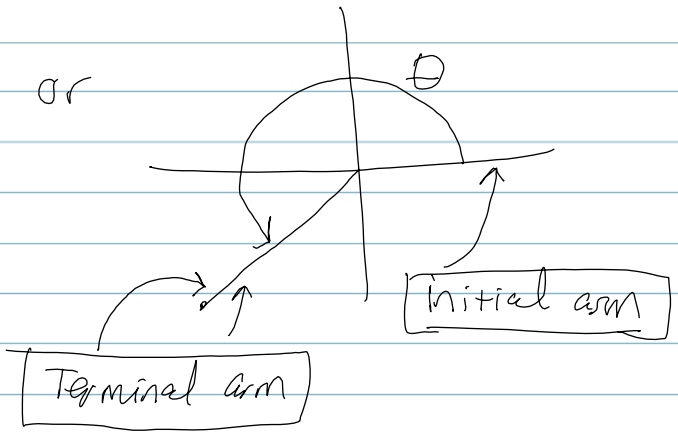
(3) Plane Trig. Draw x-y plane. 

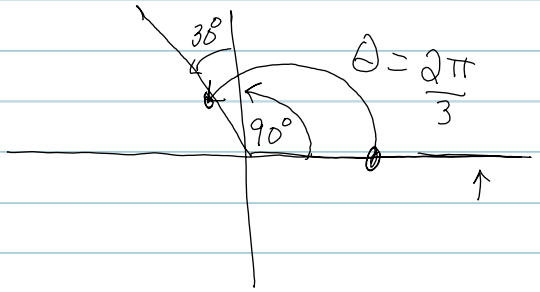
Standard Position θ is measured beginning at positive x-axis, and ending at some line.



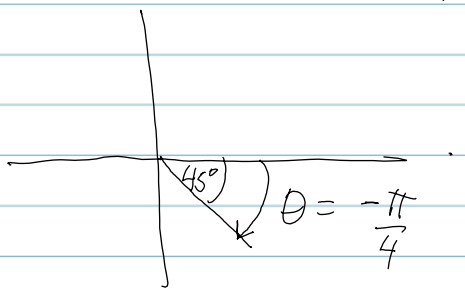
or 

θ is positive if measured counter-clockwise.
" " negative " " clockwise.

eg Draw $\theta = \frac{2\pi}{3}$ in St. pos. Note: $\frac{2\pi}{3} = \left(\frac{2}{3}\pi\right)\frac{180}{\pi} = 120°$



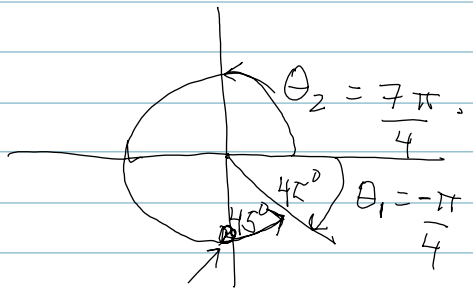
eg Draw $\theta = -\pi/4$. $\rightarrow \frac{-\pi}{4} = -45°$



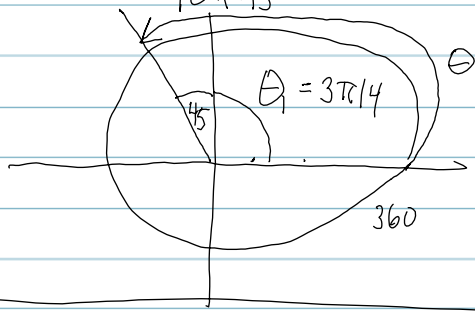
Coterminal Angles - If they begin and end on same lines.

eg $\theta_1 = -\pi/4$, $\theta_2 = 7\pi/4$ are coterminal!
$= -45°$ $= 315°$.

$315 = 270 + 45°$



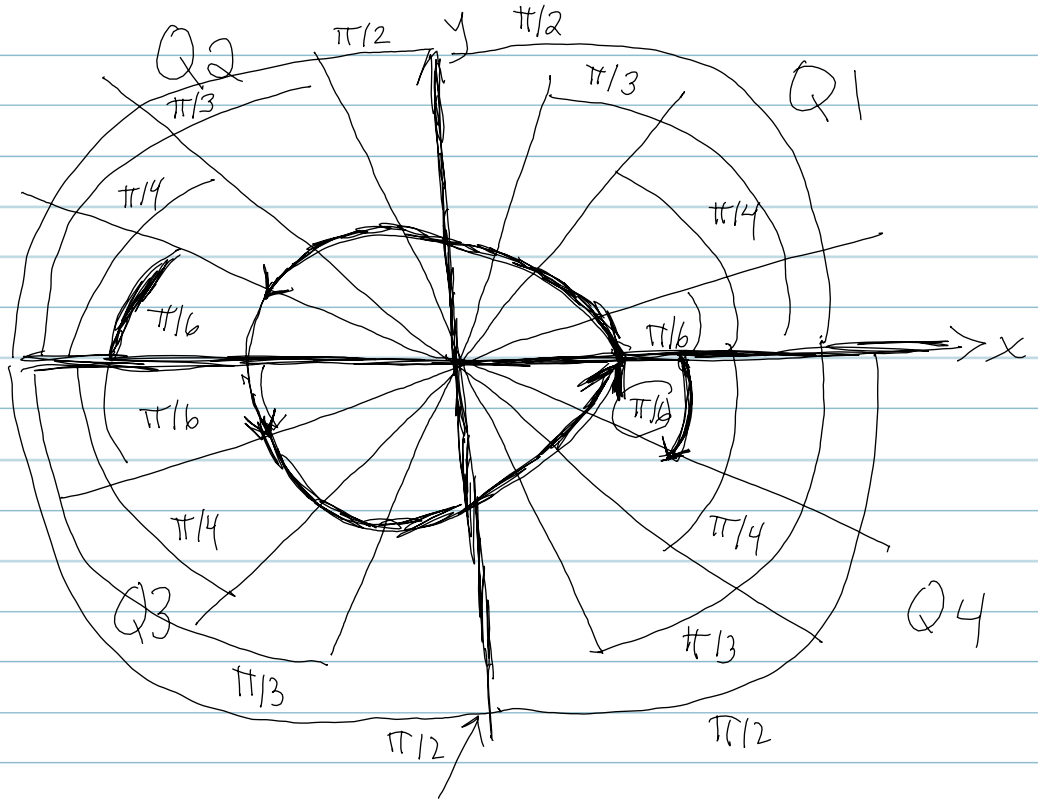
eg Are $\theta_1 = 3\pi/4$ and $\theta_2 = 11\pi/4$ coterminal?
$= 135°$ $= 495°$.
$90+45$ $= 360° + 135$



Special Angles

$\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2$. $\rightarrow$ Q1

Also, all angles above & below all x-axes.



Standard Position Measures

Q1: $0, \pi/6, \pi/4, \pi/3, \pi/2$.

Q2: Ends at π.

$\pi/6 \rightarrow \pi - \pi/6 = 5\pi/6$

$\pi/4 \rightarrow \pi - \pi/4 = 3\pi/4$

$\pi/3 \rightarrow \pi - \pi/3 = 2\pi/3$

Q3: Begins at π

$\pi/6 \rightarrow \pi + \pi/6 = 7\pi/6$

$\pi/4 \rightarrow \pi + \pi/4 = 5\pi/4$

$\pi/3 \rightarrow \pi + \pi/3 = 4\pi/3$

$\pi/2 \rightarrow \pi + \pi/2 = 3\pi/2$

Q4: ~~Begins~~ Ends at 2π

$\pi/6 \rightarrow 2\pi - \frac{\pi}{6} = 11\pi/6$

$\pi/4 \rightarrow 2\pi - \pi/4 = 7\pi/4$

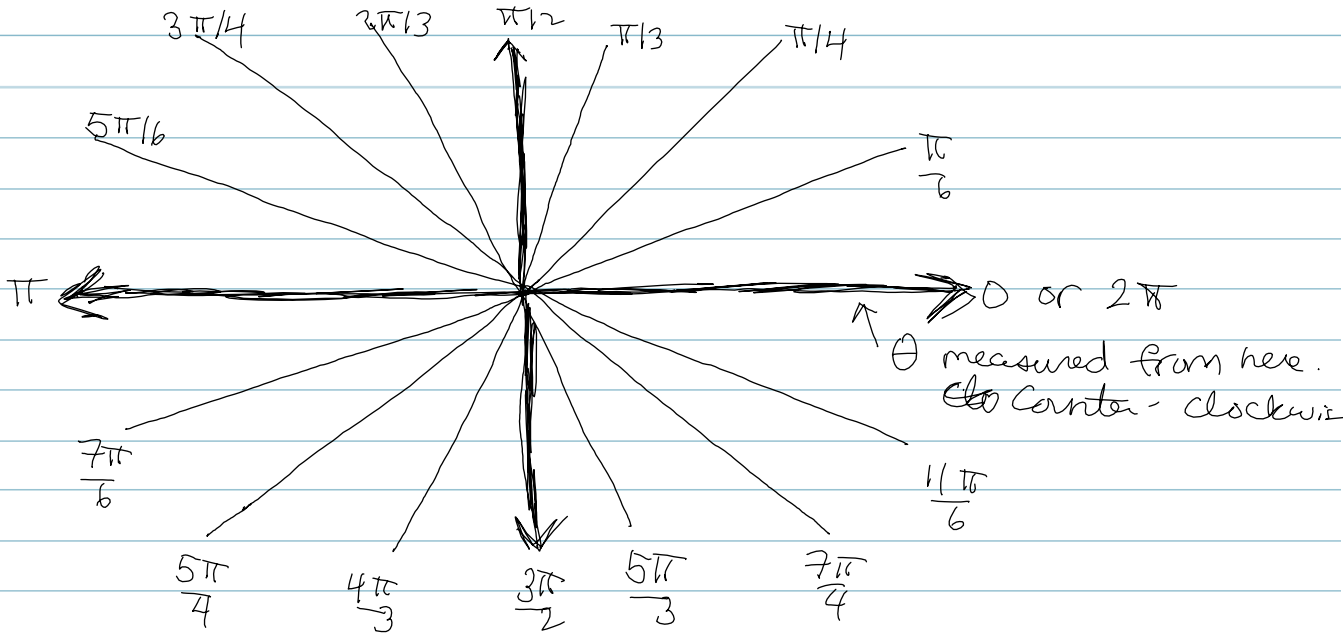
$\pi/3 = 2\pi - \pi/3 = 5\pi/3$

$2\pi \phi - \frac{\pi}{2} = 3\pi/2$

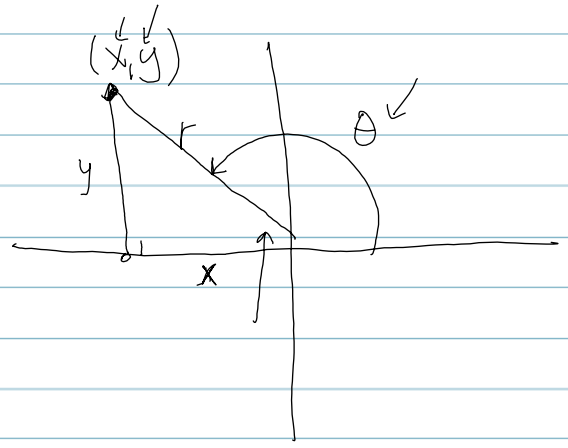
Special Angles ←

θ (deg)	0	30	45	60	90	120	135	150	180
θ (rad)	0	$\pi/6$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

θ (deg)	210	225	240	270	300	3..	33..	360
θ (rad)	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π



Trig



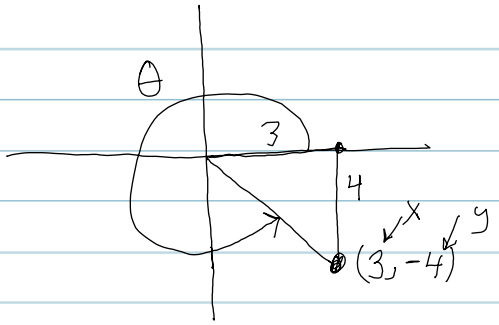
$\sin(\theta) = y/r$

$\cos(\theta) = x/r$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = y/x$

$r = \sqrt{x^2+y^2}$

eg



Find $\sin\theta$, $\cos\theta$, $\tan\theta$.

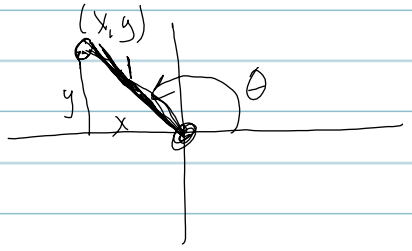
$\sin(\theta) = \frac{y}{r}$. $r = \sqrt{3^2+(-4)^2} = 5$

$= \boxed{\frac{-4}{5}}$

$\cos(\theta) = \frac{x}{r} = \frac{3}{5}$. $\tan(\theta) = \frac{y}{x} = \frac{-4}{3}$

Unit Circle – Ø · Trig Values of Special Angles

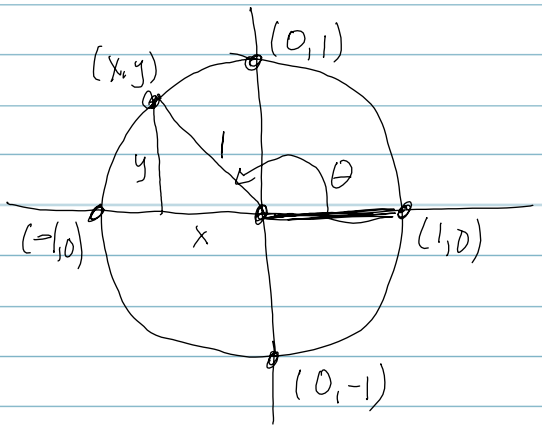
Let $r = 1$ in



Now. $\sin\theta = \frac{y}{r} = y$ ($r = 1$)

$\cos\theta = \frac{x}{r} = x$ ($r = 1$)

Along a circle of $r = 1$, y-coord $= \sin(\theta)$
x-coord $= \cos(\theta)$



Circle with $r = 1$
$\sin(\theta) = y$
$\cos(\theta) = x$

Values of Sin, Cos in Q1

Easy way to remember: All look like $\frac{\sqrt{\ }}{2}$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos(\theta) = x$	$\sqrt{4}/2 = 1$	$\sqrt{3}/2$	$\sqrt{2}/2 = 1/\sqrt{2}$	$\sqrt{1}/2 = 1/2$	$\sqrt{0}/2 = 0$
$\sin(\theta) = y$	$\sqrt{0}/2 = 0$	$\sqrt{1}/2 = 1/2$	$\sqrt{2}/2 = 1/\sqrt{2}$	$\sqrt{3}/2$	$\sqrt{4}/2 = 1$

MEMORIZE THESE!

Sin, Cos of special angles in other quadrants

(1) Realize that each is a measurement of $\pi/6$, $\pi/4$, $\pi/3$ away from + or − x-axis, and these are symmetric

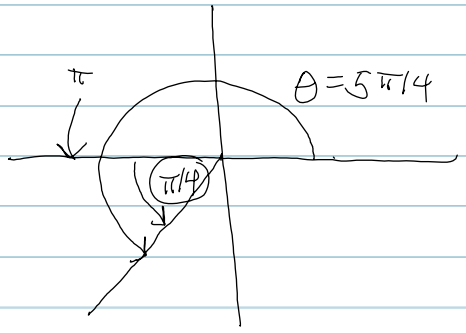
(2) Since $x = \cos(\theta)$, $y = \sin(\theta)$ → Values of $\cos(\theta)$ & $\sin(\theta)$ are going to be + or − the one values @ for Q1.

Q2 S	Q1 A
$x<0$, $y>0$	$x>0$, $y>0$
$\cos \to -$, $\sin \to +$	→ $\cos \to +$ & $\sin \to +$
Q3 N	Q4 C
$x<0$, $y<0$	$x>0$, $y<0$
$\cos \to -$, $\sin \to -$	$\cos \to +$, $\sin \to -$

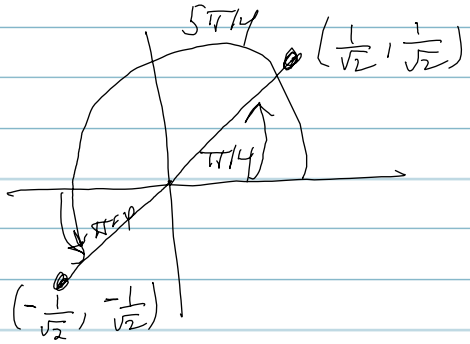
eg Find $\sin(\theta)$ and $\cos(\theta)$ for $\theta = 5\pi/4$.

$= 225° = 180 + 45$

$\pi + \pi/4$.



Pretend it's in Q1:

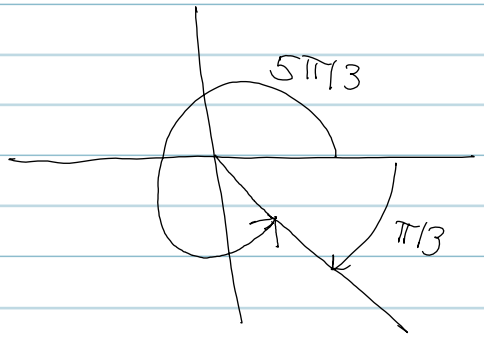


$\sin(\pi/4) = \boxed{\frac{1}{\sqrt{2}}}$, $\cos(\pi/4) = \boxed{\frac{1}{\sqrt{2}}}$. But for $5\pi/4$, it's in Q3 ie, cos → −, sin → −. By symmetry

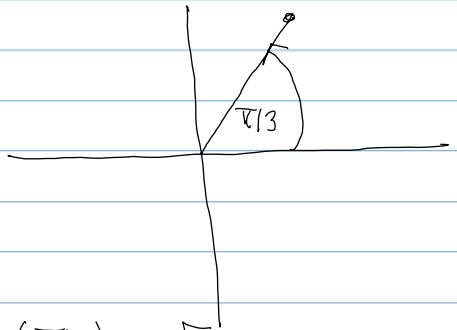
$\Rightarrow \boxed{\sin(5\pi/4) = -\frac{1}{\sqrt{2}}, \quad \cos(5\pi/4) = -\frac{1}{\sqrt{2}}}$

eg Find $\sin(5\pi/3)$ and $\cos(5\pi/3)$

Note: $\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$



Pretend it's $\pi/3$:



$\cos(\pi/3) = 1/2$, $\sin(\pi/3) = \frac{\sqrt{3}}{2}$.

But $5\pi/3$ is in Q4 $\rightarrow$ cos $\rightarrow +$, sin $\rightarrow -$

$\Rightarrow \boxed{\cos(5\pi/3) = 1/2 \text{ , } \sin(5\pi/3) = -\frac{\sqrt{3}}{2}}$

Special Angles

Q1

θ	Boundary 0	$30°$ $\pi/6$	$45°$ $\pi/4$	$60°$ $\pi/3$	Boundary $90°$ $\pi/2$
$x = \cos(\theta)$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$y = \sin(\theta)$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1

Q2

θ	$120°$, $2\pi/3$	$135°$, $3\pi/4$	$150°$, $5\pi/6$	Boundary $180°$, π
$x = \cos\theta$	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1
$y = \sin\theta$	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

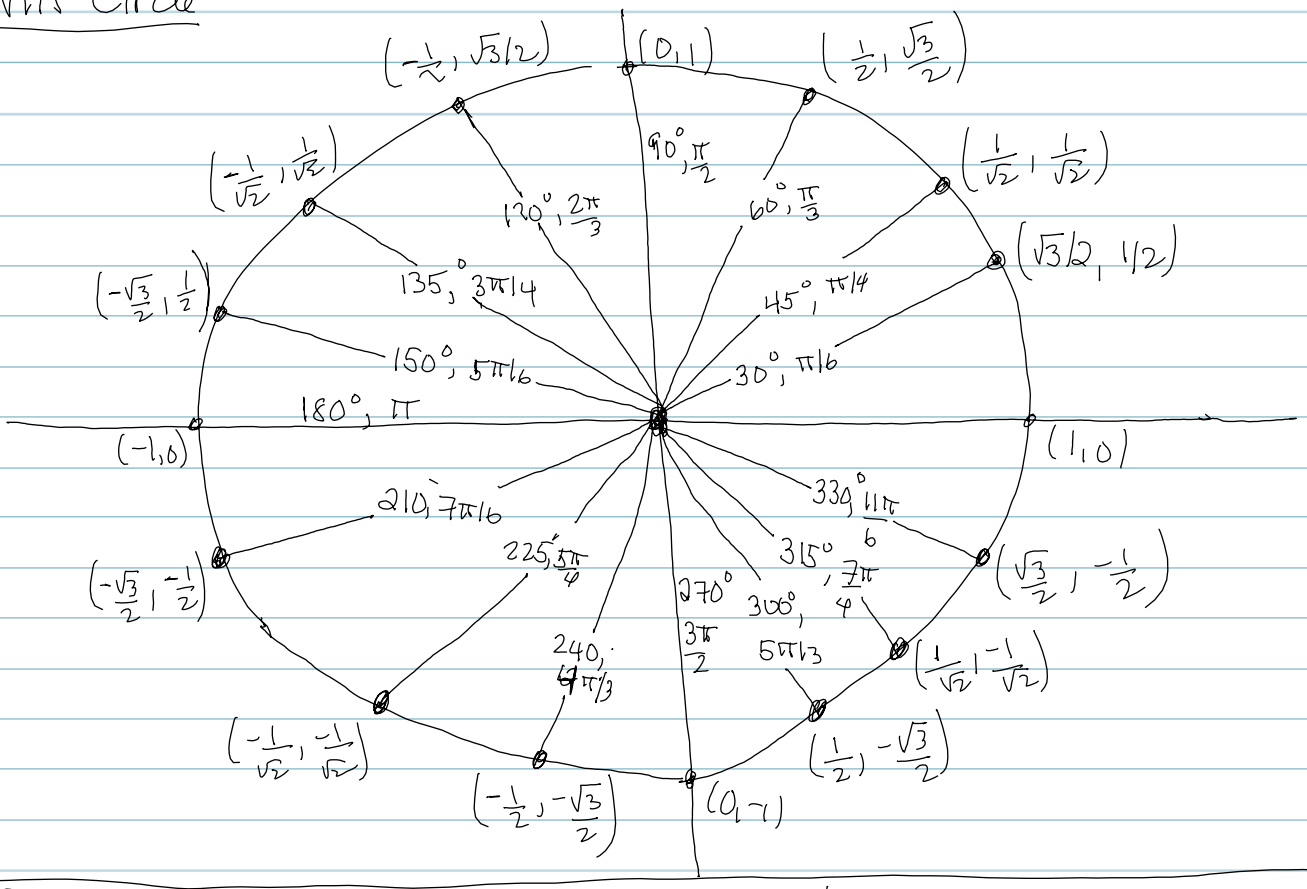
Q3

θ	$210°$, $7\pi/6$	$225°$, $5\pi/4$	$240°$, $4\pi/3$	Boundary $270°$, $3\pi/2$
$x = \cos\theta$	$-\sqrt{3}/2$	$-1/\sqrt{2}$	$-1/2$	0
$y = \sin\theta$	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1

Q4

θ	300, $5\pi/3$	315, $7\pi/4$	330, $11\pi/6$	Boundary 360, 2π
$x=\cos(\theta)$	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$y=\sin(\theta)$	$-\sqrt{3}/2$	$-1/\sqrt{2}$	$-1/2$	0

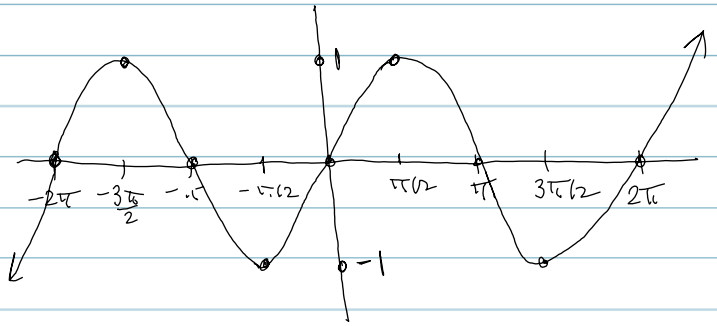
Unit Circle



Graphs of Trig Functions Sin, Cos, tan.

① $y=\sin(x)$

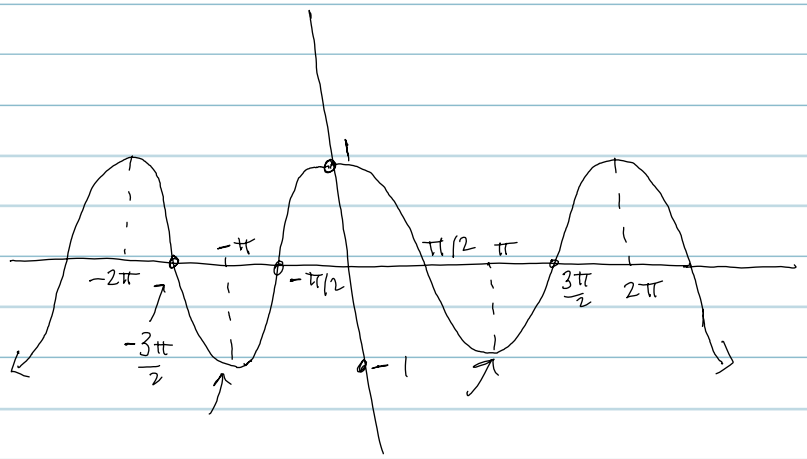
x	-2π	$-3\pi/2$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
y	0	1	0	-1	0	1	0	-1	0



Domain: $\mathbb{R}$, $(-\infty,\infty)$

Range: $[-1,1]$

$y = \cos(x)$



$D = \mathbb{R}$, or $(-\infty, \infty)$
$R = [-1, 1]$

Notice: ① Cos is mirrored in y-axis
⇒ cos is an even function
$\cos(x) = \cos(-x)$ for all x.

② Sin is mirrored in both X & y axis.
⇒ Sin is an odd function
$\sin(x) = -\sin(-x)$

③ $\cos(x)$ is $\sin(x)$ shifted left by $\pi/2$.

Horizontal shift: To shift $f(x)$ left or right you do $f(x \mp a)$ ← shift

$f(x-a)$ → Right by a
$f(x+a)$ → Left " a

$f(x) = \sin(x)$ Shift left by $\pi/2$.
$f(x+\pi/2) = \sin(x+\pi/2)$
But this is cos!

⇒ $\boxed{\cos(x) = \sin(x+\pi/2)}$

$\sin^2(x) + \cos^2(x) = 1$, $1 + \tan^2(x) = \sec^2(x)$.

$y = \tan(x)$. Start in $-\pi/2$ to $\pi/2$.

$y = \frac{\sin(x)}{\cos(x)}$ → Undefined when $\cos(x) = 0$. ie,

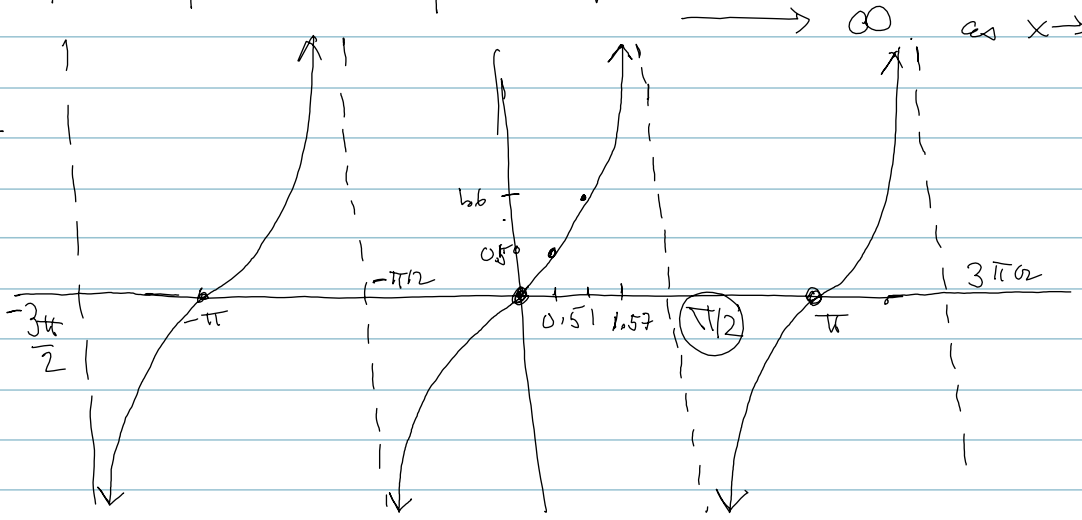
$\boxed{x = \pm \pi/2,\ \pm 3\pi/2,\ \pm 5\pi/2}$

In general: $x = \pm(\text{odd }\#)(\pi/2)$

$\Rightarrow$ Asymptotes at $\pm(\text{odd})(\pi/2) \approx \pm(\text{odd})(1.5708)$

x (rad)	0	0.5	1	1.5	1.57	1.5707
$\tan(x)$	0	0.5	1.6	14.1	1255.8	10381.3

$\frac{\sin(0)}{\cos(0)} = \frac{0}{1}$



D: $\{x: x \neq (\text{odd }\#)\,\pi/2\}$

R: $\mathbb{R}$ or $(-\infty, \infty)$.