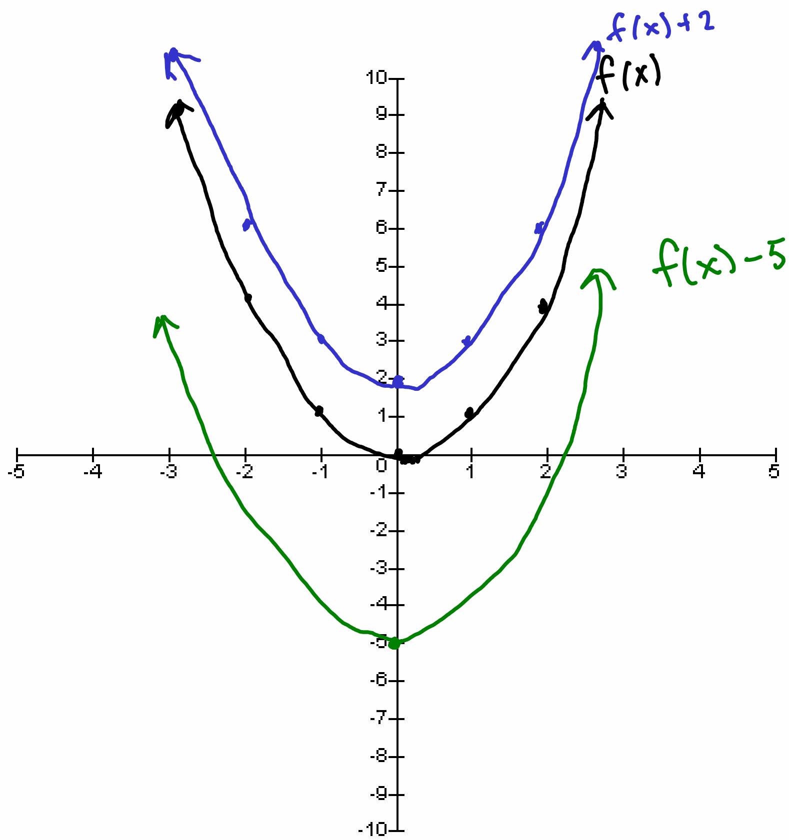
Shifting Graphs

This question deals with shifting the function $f(x) = x^2$, but the results generalize to any function.

1. Draw the graph of $f(x) = x^2$.

Table of values:

x	-3	-2	-1	0	1	2	3
f(x)	9	4	1	0	1	4	9



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2. On the above axes, draw the graphs of $f(x)+2$ and $f(x)-5$.

Table of values for $f(x)+2$: $f(x) = x^2$.

x	-3	-2	-1	0	1	2	3
f(x) + 2	11	6	3	2	3	6	11

x^2+2

Table of values for $f(x)-5$:

x	-3	-2	-1	0	1	2	3
f(x) – 5							

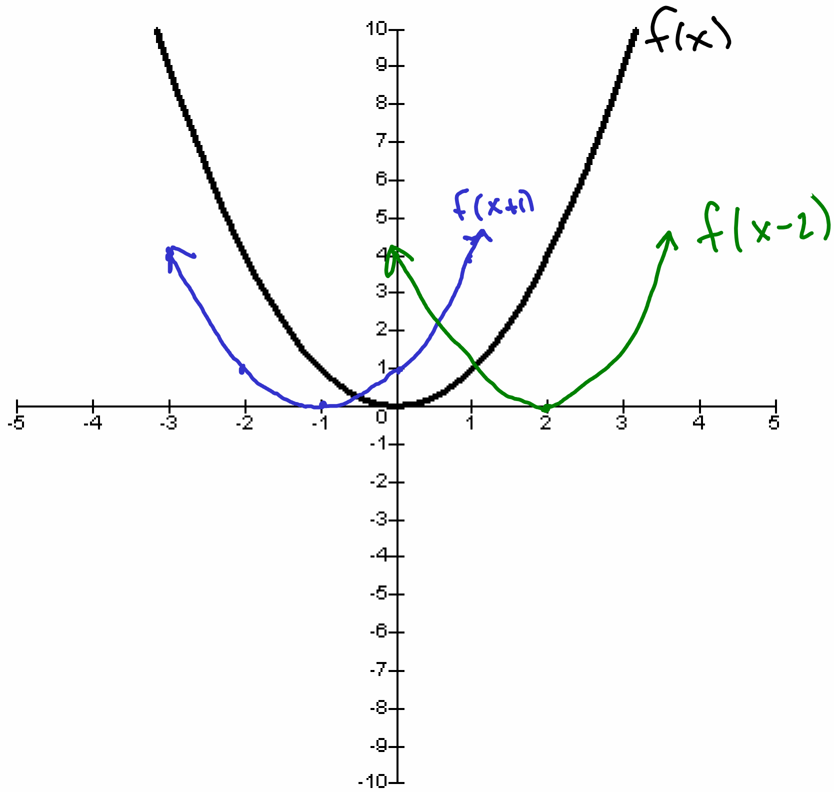
IN GENERAL, for ANY function $f(x)$**:**

y + K

1. $f(x)+k$ moves the graph of $f(x)$ ___up___ by ___K___ units.

2. $f(x)-k$ moves the graph of $f(x)$ ___down___ by ___K___ units.

Again, we're dealing with the function $f(x) = x^2$, which is drawn again here:



3. Draw the graphs of $f(x+1)$ and $f(x-2)$.

$f(x) = x^2$

Table of values for $f(x+1)$:

x	-3	-2	-1	0	1	2	3
f(x + 1)	f(-2) = 4	f(-1) = 1	f(0) = 0	f(1) = 1	4	9	16

Table of values for $f(x-2)$:

x	-3	-2	-1	0	1	2	3
f(x – 2)							

IN GENERAL, for ANY function $f(x)$:

1. $f(x+k)$ moves the graph of $f(x)$ to the Left by K units.
2. $f(x-k)$ moves the graph of $f(x)$ to the right by K units.

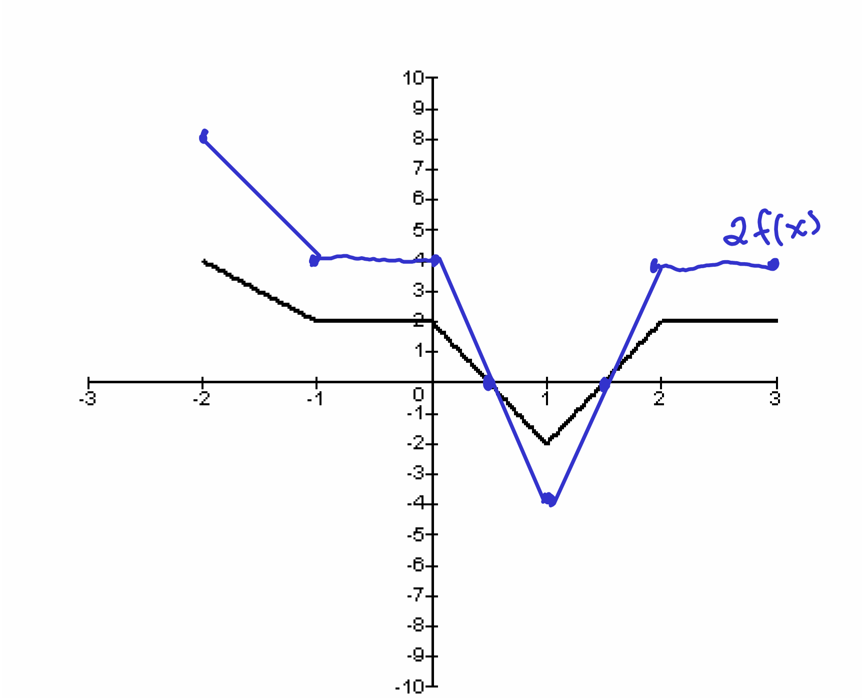
Summary

1. **Vertical shifting:** $f(x)+k$ or $f(x)-k$; i.e., addition/subtraction of k is OUTSIDE the brackets of $f(x)$.
2. **Horizontal shifting:** $f(x+k)$ or $f(x-k)$; i.e., addition/subtraction of k is INSIDE the brackets of $f(x)$.

Note that all this applies to ANY function $f(x)$, not just for the example of x^2 we considered here!

Scaling Graphs

This question deals with the function $f(x)$, which has the following graph:



2y

1. On the above axes, draw the graph of $2f(x)$. (It might be helpful to make a table of values for $f(x)$.)

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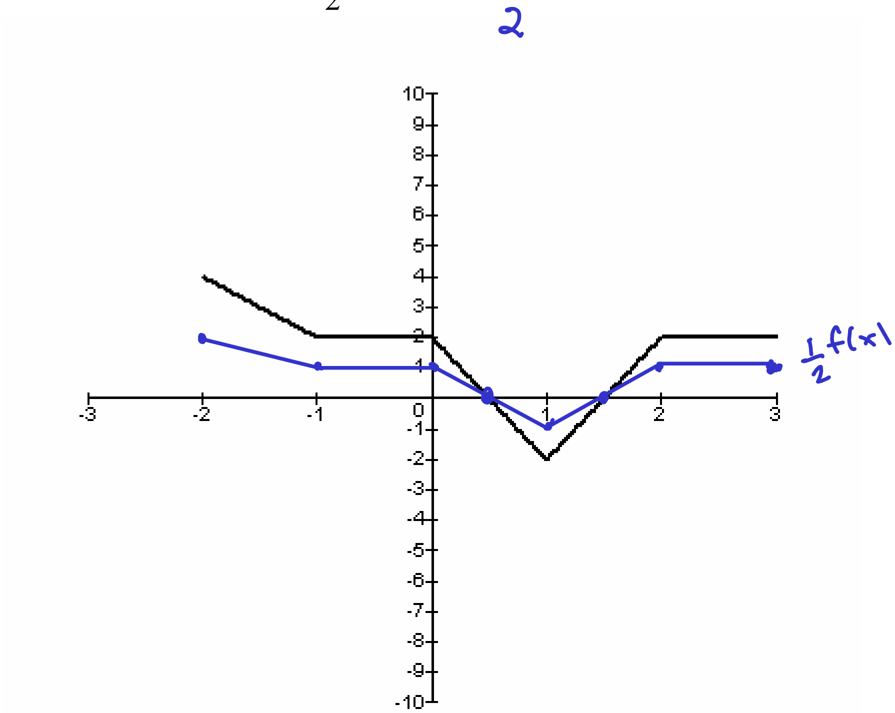
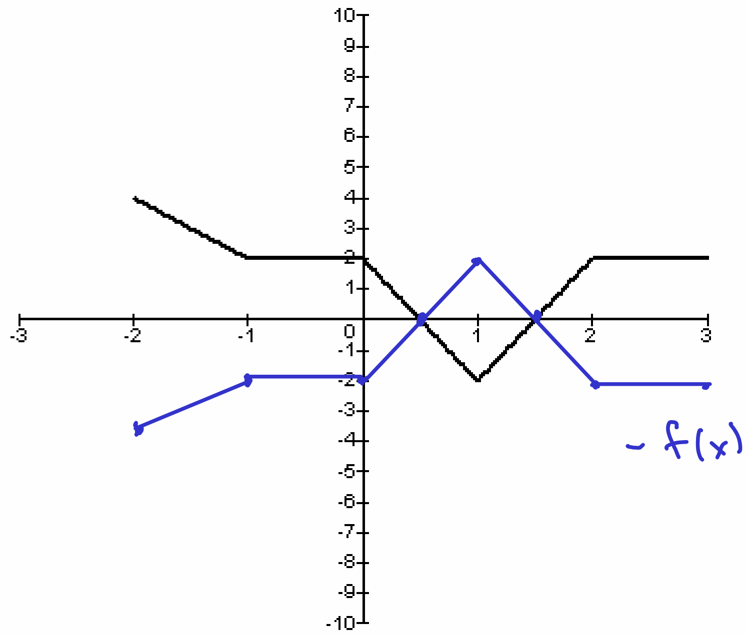
2. On the following axes, draw the graph of $\frac{1}{2}f(x)$. $\frac{1}{2}y$



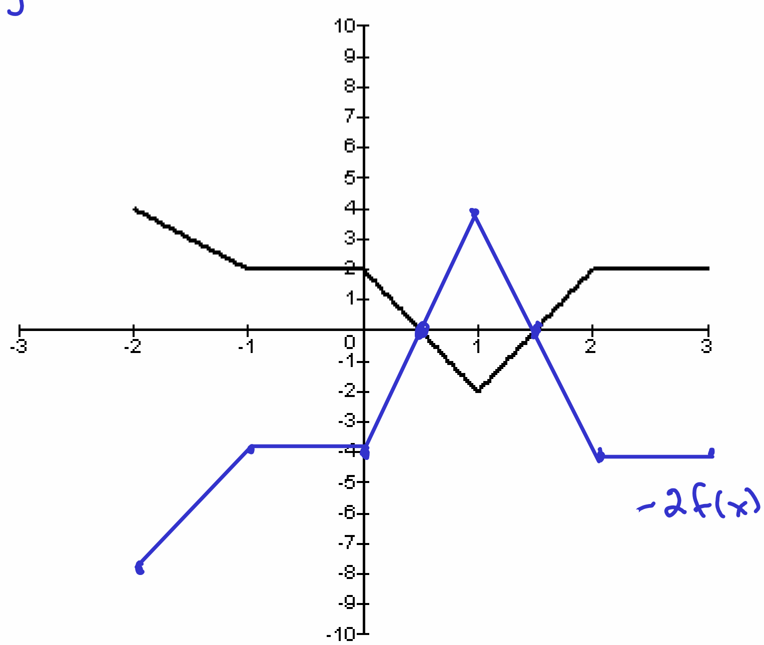
3. Draw the graph of $-f(x)$.

$-y$



4. Draw the graph of $-2f(x)$.

$-2y$



IN GENERAL: For ANY function $f(x)$:

1. If $k>1$, then $kf(x)$ stretches the graph of $f(x)$ by a factor of K.

2. If $0<k<1$, then $kf(x)$ compresses the graph of $f(x)$ by a factor of K.

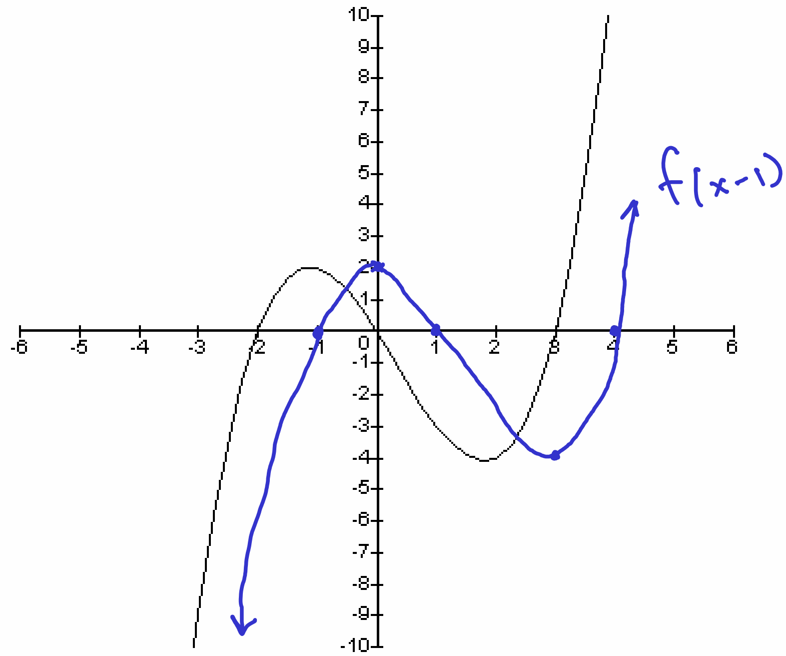
3. $-kf(x)$ first stretch / compress the graph of $f(x)$, then reflect it about the X axis.

Shifting and Scaling Example

The graph of a function is given. Draw the graph of the function resulting from the following:

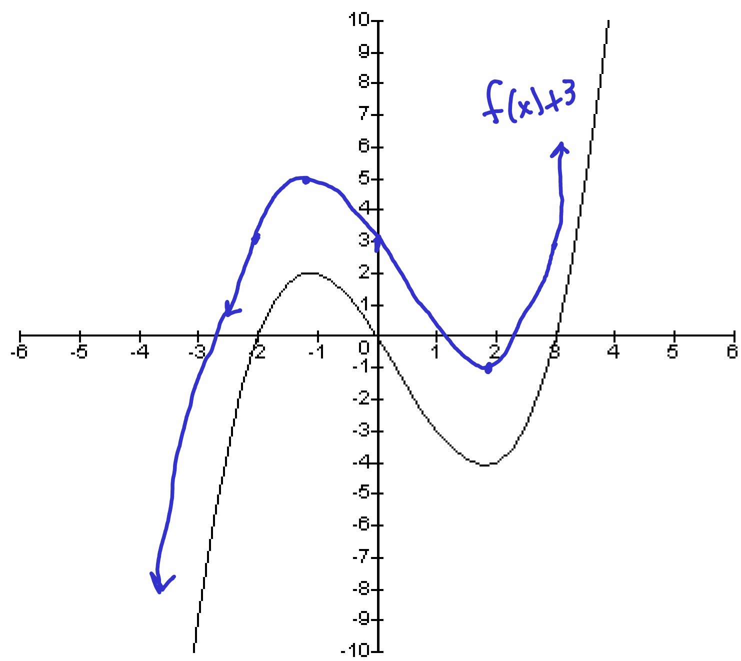
1. $f(x-1)$

Right 1



2. $f(x)+3$

up 3

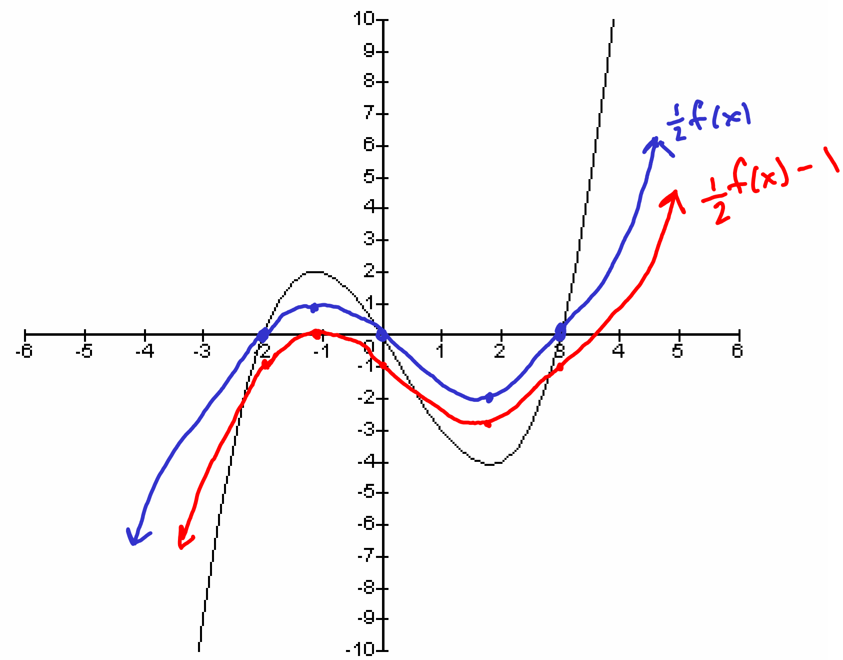


3. $\frac{1}{2}f(x)-1$

BEDMAS

① $\frac{1}{2}f(x)$

② $\frac{1}{2}f(x)-1$



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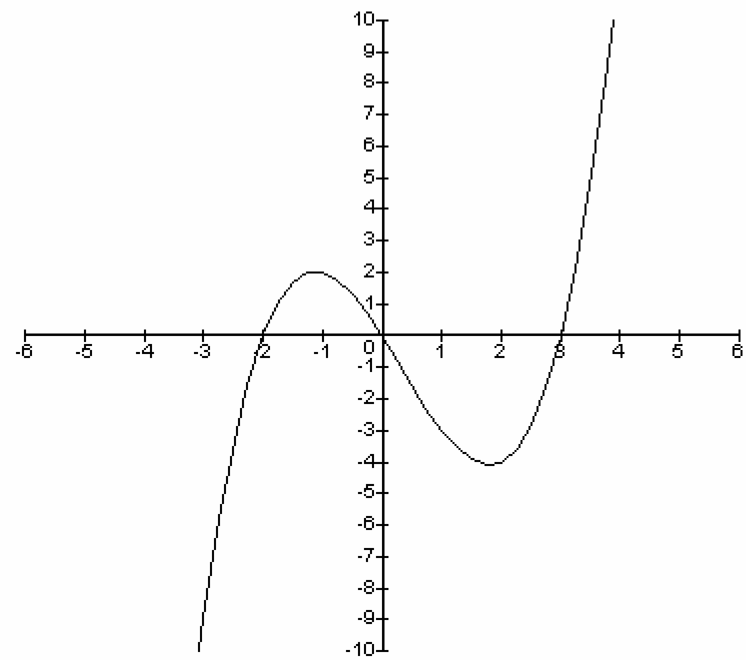
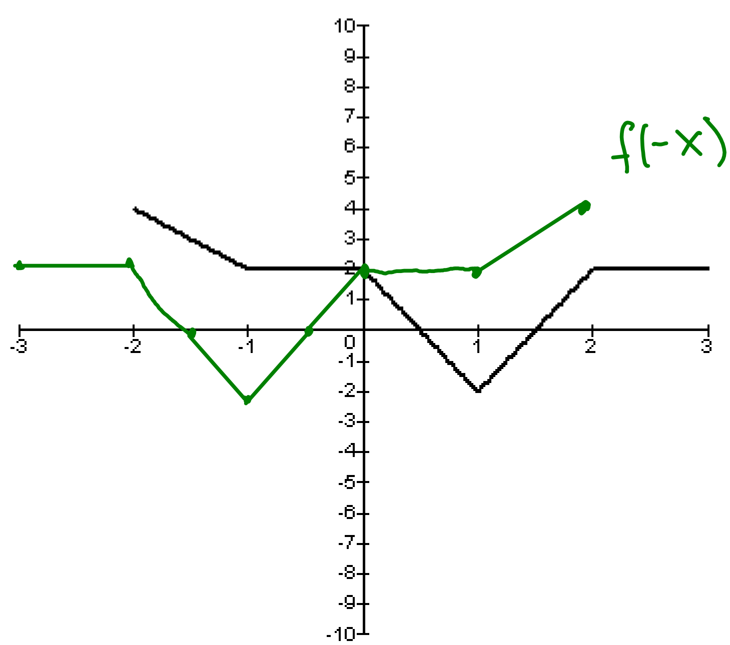
4. Tough one: $1-2f(x+3)$

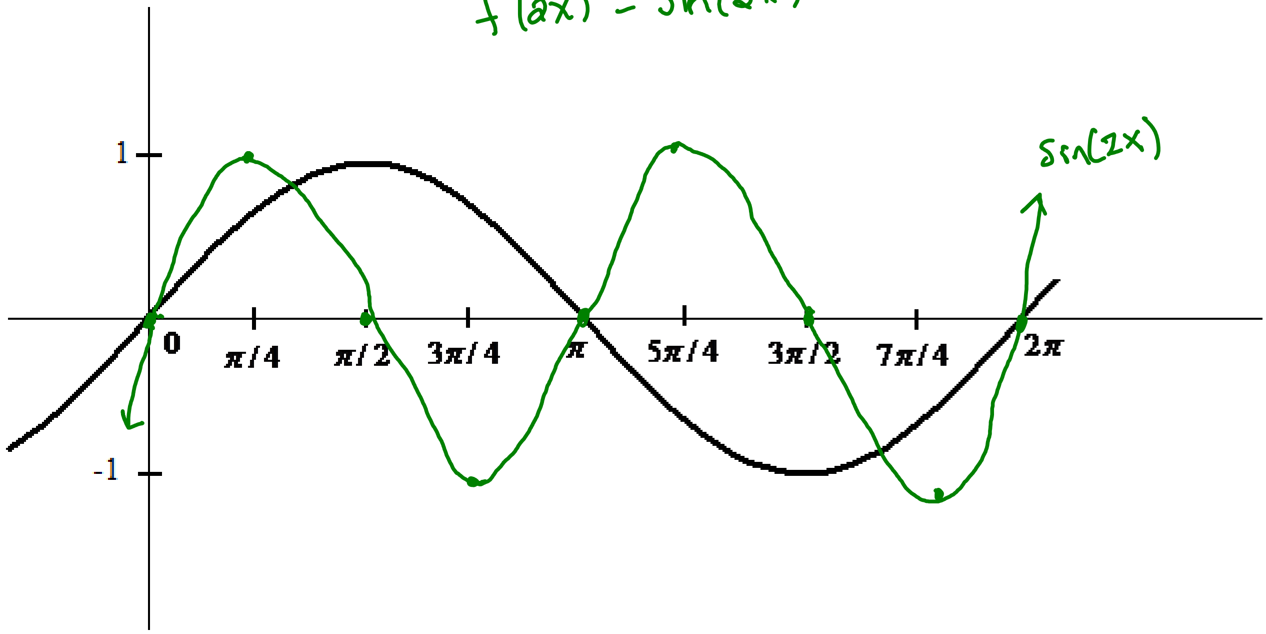


Draw the graph of $f(-x)$.



Consider the graph of $f(x) = \sin(x)$:

$f(2x) = \sin(2x)$



1. On the same axes above, draw the graph of $\sin(2x)$ (although not necessary, it might be helpful to use the following table of values):

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
~~$\sin(x)$~~	~~0~~	~~0.7~~	~~1~~	~~0.7~~	~~0~~	~~- 0.7~~	~~- 1~~	~~- 0.7~~	~~0~~
$\sin(2x)$	$\sin(2\cdot 0)=0$	$\sin(\pi/2)=1$	0	-1	0	1	0	-1	0

IN GENERAL: *For ANY function* $f(x)$: *If* $c > 1$, *then*

1. $cf(x)$ Vertically Stretches the graph of $f(x)$ by a factor of c.
2. $f(x)/c$ " Compresses the graph of $f(x)$ by a factor of c.
3. $f(cx)$ horizontally compress the graph of $f(x)$ by a factor of c.
4. $f(x/c)$ " stretch the graph of $f(x)$ by a factor of c.
5. $-f(x)$ reflects the graph through the x axis.
6. $f(-x)$ " the graph through the y axis.