Logarithms

eg $3^2 = ?$ Asks "given base (3) and exponent (2), what is the result?"
Ans is 9 → $3^2 = 9$.

eg $3^? = 9$ Asks "Base is 3, result is 9. What was the exp?". Ans is 2 ($3^2 = 9$)

This is a logarithm: $3^? = 9$ asks the same thing as $\log_3(9) = ?$

ie $\log_3(9)$ asks "What power of 3 produces 9",
Ans = 2 b/c $3^2 = 9$.

$\Rightarrow \log_3(9) = 2$.

General: $\log_a(x)$ asks "What power of a gives x?"
ie same Q as $a^? = x$.

$$\text{If } y = \log_a(x) \iff a^y = x$$

$a^? = x$

The power of a that gives x is y.

$\boxed{\text{Log is the inverse of an exponential function}}$

eg $\log_2(32)$? "What power of 2 produces 32"
Ans is 5 $2^5 = 32$.

$\boxed{\log_2(32) = 5}$

eg $\log_3(13)$? "$3^? = 13$" $3^2 = 9$, $3^3 = 27$ ⇒
Too low, Too high
⇒ Ans b/w 2 & 3.

Calculator: $\boxed{\log_{10} \to \log}$
$\log_e \to \ln$

Formula: $\log_a(x) = \dfrac{\log_{10}(x)}{\log_{10}(a)} = \dfrac{\log(x)}{\log(a)}$

Here: $\log_3(13) = \dfrac{\log(13)}{\log(3)} = \boxed{2.334717519}$

ie, $3^{2.334717519}$ "=" 13

Base e (natural base)

$e \approx \underline{2.718281828\ldots}$ (Best base for use in calculus).

$\log_e(x) \longrightarrow \ln(x)$ (log natural)

$\ln(x) \to$ " What power of e produces x " $e^{?} = x$.

eg $\ln(e^3) \to$ " What power of e produces e^3 ?
$e^{?} = e^3$ clearly 3!
$\Rightarrow \ln(e^3) = 3$.

eg $\ln(5) \to$ " $e^{?} = 5$ ". Calc. has ln button.
$\ln(5) = 1.609437912$

ie $e^{1.609437912}$ "=" 5

Graphs of log functions

$\boxed{y = \log_a(x) \text{ is inverse of } y = a^x}$

$a > 0$ and $a \neq 1$.

Why?

1. if $a < 0$, a^x wouldn't always be defined.

   eg $a = -4$

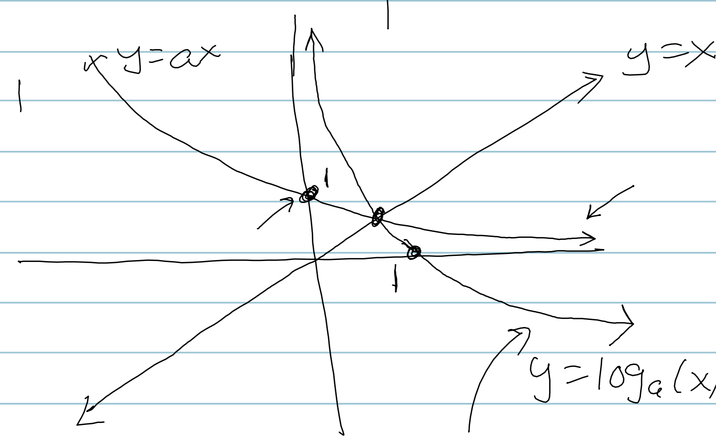
   $y = (-4)^x$ if $x = 1/2$

   $y = (-4)^{1/2} = \sqrt{-4} \to$ und.

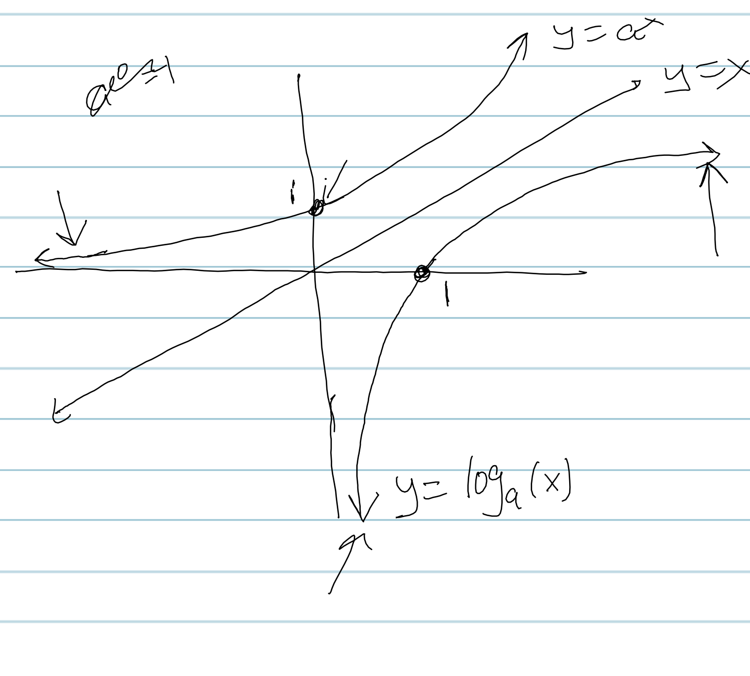
2. $a \neq 1$: If $a = 1$, $y = a^x = 1^x = 1$

   $y = 1$

Two cases (1) $0 < a < 1$



(2) $a > 1$



Domain $(0, \infty)$

Range $(-\infty, \infty) = \mathbb{R}$.

Properties of logs (includes ln)

① $\log_a(a^x) = x$ (for all x) $\qquad a^{?} = a^x$

& $a^{\log_a(x)} = x$ $(x > 0)$

② $\log_a(xy) = \log_a(x) + \log_a(y)$

③ $\log_a(x/y) = \log_a(x) - \log_a(y)$

④ $\log_a(x^p) = p \cdot \log_a(x)$

⑤ $\log_a(a) = 1$, $\log_a(1) = 0$

$a^{?} = 1 \qquad a^0 = 1$

$a^{?} = a^1$

eg Express as a single log.

① $4\log_3(2) + 7\log_3(5)$

$= \log_3(2^4) + \log_3(5^7)$ (P4)

$= \log_3(2^4 \cdot 5^7)$ (P2)

② $\ln(5) - 5\ln(3)$

$\ln(5) - \ln(3^5)$ (P4)

$\ln(5/3^5)$ (P3)

eg Solve $3^x = 21$

① Take log (any base) of both sides.
(say $\log_3$).

$\log_3(3^x) = \log_3(21)$

(P1) $\rightarrow$ $\boxed{x = \log_3(21)}$ $\leftarrow$

or $x = \dfrac{\log(21)}{\log(3)} = 2.771243749.$

eg Solve $e^{7-4x} = 6$.

① $\ln(e^{7-4x}) = \ln(6)$ $\qquad \log_a(a^x) = x$

(P1) $\quad 7 - 4x = \ln(6)$ $\qquad \log_a(a^{\sim}) = \sim$

$\log_e(e^{\sim}) = \sim$

isolate $\quad 7 - \ln(6) = 4x$ $\qquad \ln(e^{\sim}) = \sim$

$$\boxed{x = \frac{7 - \ln(6)}{4}} = 1.362060133$$

eg Solve $2(5^x) = 3(7^x)$.

① $\ln(2(5^x)) = \ln(\overset{a}{3}(\overset{b}{7^x}))$

$\ln(2) + \ln(5^x) = \ln(3) + \ln(7^x)$ $\qquad$ (P2)

$\ln(2) + x \cdot \ln(5) = \ln(3) + x \cdot \ln(7)$ $\qquad$ (P4)

$x \cdot \ln(5) - x \cdot \ln(7) = \ln(3) - \ln(2)$

$x(\ln(5) - \ln(7)) = \ln(3) - \ln(2)$

$$\boxed{x = \frac{\ln(3) - \ln(2)}{\ln(5) - \ln(7)}} = -1.205047339.$$

eg Solve $\log_3(x) = 5$. $\quad$ Raise 3 to power of each side.

$3^{\log_3(x)} = 3^5$

(P1) $\quad \boxed{x = 243}$

eg Solve $\ln(x^2 - 1) = 3$ $\qquad \ln \to \log_e$ $\quad$ Raise e to both

$e^{\ln(x^2-1)} = e^3$ $\qquad e^{\ln x} = x, \; e^{\ln(\sim)} = \sim$

(P1) $\quad x^2 - 1 = e^3 \to x^2 = e^3 + 1$

$\to x = \pm\sqrt{e^3+1}$.

Check $\quad \ln((\sqrt{e^3+1})^2 - 1) = \ln(e^3 + 1 - 1) = \ln(e^3) = 3$ ✓

$\ln((-\sqrt{e^3+1})^2 - 1) = \ldots\ldots = 3$ ✓