

Solving Quadratic Equations:  $ax^2 + bx + c = 0$

Two methods:

1. Quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. Factoring into the form  $(mx + n)(px + q) = 0$

1. Quadratic formula

Eg  $2x^2 - 3x - 2 = 0$

$a=2, b=-3, c=-2$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{25}}{4} = \frac{3 \pm 5}{4} \quad \textcircled{1} \frac{3+5}{4} = \boxed{2}, \quad \textcircled{2} \frac{3-5}{4} = \boxed{\frac{-1}{2}}$$

Eg  $5x^2 + x - 4 = 0$   $x = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-4)}}{2(5)} = \frac{-1 \pm \sqrt{81}}{10}$

$$= \frac{-1 \pm 9}{10} \Rightarrow \textcircled{1} \frac{-1+9}{10} = \boxed{\frac{4}{5}} \quad \textcircled{2} \frac{-1-9}{10} = \boxed{-1}$$

② Factoring Logic:  $yz = 0 \Rightarrow y=0$  or  $z=0$

How to factor  $ax^2 + bx + c = 0$

Step 1: Find numbers that MULTIPLY together to produce  $ac$ .

Step 2: Of the pairs of numbers in step 1, find the pair that ADDS to give  $b$ .

Eg  $2x^2 - 3x - 2 = 0$

①  $ac = (2)(-2) = -4$ . Factors:  $(1)(-4)$ ,  $(-1)(4)$ ,  $(2)(-2)$

② Find the pair that add to  $-3$ :  $1$  and  $-4$

Write the  $-3x$  as  $1x$  and  $-4x$   $-3x \Rightarrow x - 4x$

$$2x^2 - 3x - 2 = \underline{2x^2 + x} - \underline{4x - 2} = \underbrace{x(2x+1)} - \underline{2(2x+1)}$$

$$\boxed{2x^2 - 3x - 2 = (2x+1)(x-2)}$$

$$2x^2 - 3x - 2 = 0$$

$$(2x+1)(x-2) = 0 \Rightarrow 2x+1 = 0 \quad \text{or} \quad x-2 = 0$$

$$2x = -1$$

or

$$\boxed{x = 2}$$

$$\boxed{x = -1/2}$$

eg  $5x^2 + x - 4 = 0$

①  $ac = 5(-4) = -20$ . Factors:  $(1)(-20), (-1)(20), (-2)(10), (10)(-2), (4)(-5), (-4)(5)$

② Add to  $b = 1$ : Use  $-4$  and  $5$   
 $x \rightarrow -4x + 5x$

$$5x^2 + x - 4 = \underline{5x^2 - 4x} + \underline{5x - 4}$$

$$= x(5x-4) + (5x-4) = (5x-4)(x+1)$$

$$\Rightarrow 5x^2 + x - 4 = (5x-4)(x+1) = 0$$

$$5x-4=0$$

$$\text{or } x+1=0$$

$$\boxed{x = 4/5} \quad \boxed{x = -1}$$

eg  $3x^2 - 14x + 8 = 0$

①  $ac = 24$ .  $1(24), (-1)(-24), (2)(12), (-2)(-12)$

②  $b = -14$  Use  $-2$  and  $-12$

Write  $-14x$  as  $-2x - 12x$

$$\Rightarrow 3x^2 - 14x + 8 = \underline{3x^2 - 2x} - \underline{12x + 8} = x(3x-2) - 4(3x-2)$$

$$= (3x-2)(x-4)$$

$$\Rightarrow 3x^2 - 14x + 8 = (3x-2)(x-4) = 0$$

$$\Rightarrow 3x-2=0$$

or

$$x-4=0$$

$$\boxed{x = 2/3}$$

or

$$\boxed{x = 4}$$