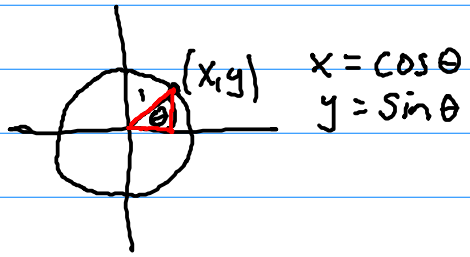
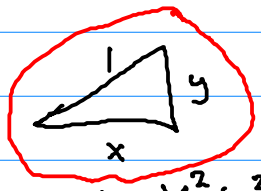
Trigonometric Identities

① $\cos^2\theta + \sin^2\theta = 1$ Why?





$\Rightarrow x^2 + y^2 = 1$

$\Rightarrow (\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$ //

② $\sec^2\theta - \tan^2\theta = 1$ Why?

$\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta}$ Know: $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1$ //

③ $\sin(a \pm b) = \sin(a)\cos(b) \pm \sin(b)\cos(a)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

Double Angle Formulas

④ $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

Why? $\sin(2\theta) = \sin(\theta+\theta) = \sin(\theta)\cos(\theta) + \sin(\theta)\cos(\theta)$

$= 2\sin(\theta)\cos(\theta)$ //

⑤ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ or $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$

Why? $\cos(2\theta) = \cos(\theta+\theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$

$= \cos^2(\theta) - \sin^2(\theta)$ //

Or: $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ But $\cos^2(\theta) = 1 - \sin^2(\theta)$

$= 1 - \sin^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$ //

Or $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ But $\sin^2(\theta) = 1 - \cos^2(\theta)$

$= \cos^2(\theta) - (1 - \cos^2(\theta)) = \cos^2(\theta) - 1 + \cos^2(\theta) = 2\cos^2(\theta) - 1$ //

$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$. Why? $\cos(2\theta) = 2\cos^2(\theta) - 1$

$\Rightarrow \cos(2\theta) + 1 = 2\cos^2(\theta)$

$\Rightarrow \frac{\cos(2\theta)+1}{2} = \cos^2(\theta)$ //

$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ Why? $\cos(2\theta) = 1 - 2\sin^2(\theta)$

$\Rightarrow 2\sin^2(\theta) = 1 - \cos(2\theta)$

$\Rightarrow \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$

Random Identities

eg $\sin(\theta)\cot(\theta) = \cos(\theta)$.

Why? $\sin(\theta)\cot(\theta) = \frac{\sin(\theta)\cdot\cos(\theta)}{\sin(\theta)} = \cos(\theta)$ //

eg $\cot^2(\theta) + \sec^2(\theta) = \tan^2(\theta) + \csc^2(\theta)$ Why?

$\sec^2(\theta) - \tan^2(\theta) = 1$

$\csc^2(\theta) - \cot^2(\theta) = ? = \frac{1}{\sin^2(\theta)} - \frac{\cos^2(\theta)}{\sin^2(\theta)} = \frac{1-\cos^2(\theta)}{\sin^2(\theta)} = \frac{\sin^2(\theta)}{\sin^2(\theta)} = 1$

$\Rightarrow \csc^2(\theta) - \cot^2(\theta) = 1$

$\Rightarrow \sec^2(\theta) - \tan^2(\theta) = \csc^2(\theta) - \cot^2(\theta)$

$\Rightarrow \sec^2(\theta) + \cot^2(\theta) = \csc^2(\theta) + \tan^2(\theta)$ //