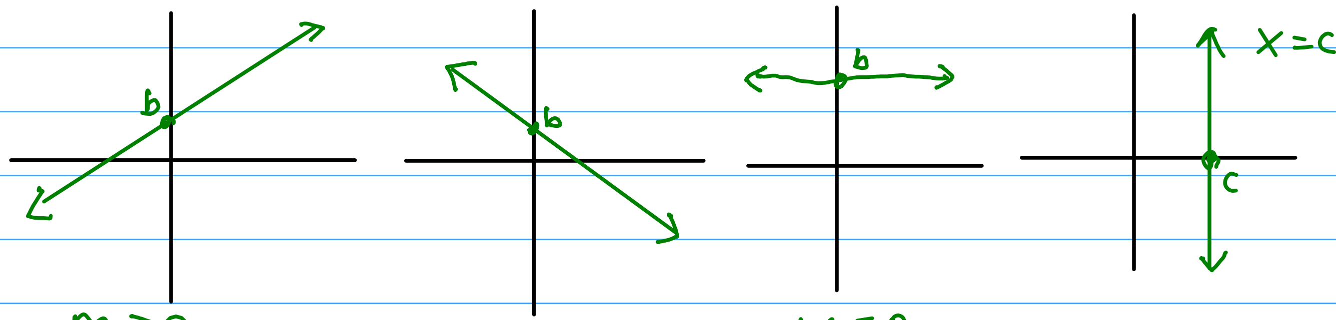
Graphs, Domain, and Range of Basic Functions

Linear: $y = mx+b$. m = slope, b = y-intercept

$m>0$: line goes up. $m<0$: line goes down. $m=0$: line is flat (horizontal)

$x=c$: line is vertical

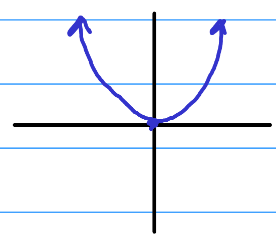


$m>0$ $m<0$ $m=0$

$D = \mathbb{R}$ or $(-\infty,\infty)$, $R = \mathbb{R}$ or $(-\infty,\infty)$

Power Functions: $y = x^n$

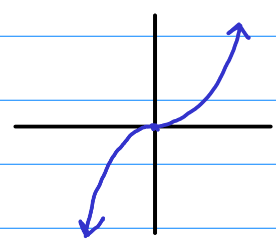
1. n is even: $x^2, x^4, \ldots$



Domain: $\mathbb{R}$

Range: $[0,\infty)$

2. n is odd: $x^3, x^5, \ldots$



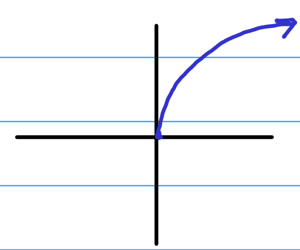
Domain: $\mathbb{R}$

Range: $(-\infty,\infty)$

3. n = 1/k, k is even (even roots):

$x^{1/2}, x^{1/4}, \ldots$

$\sqrt{x}, \sqrt[4]{x}, \ldots$



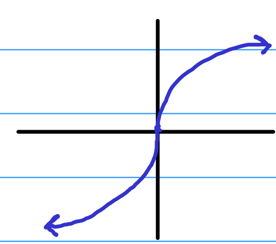
Domain: $[0,\infty)$

Range: $[0,\infty)$

4. n = 1/k, k is odd (odd roots):

$x^{1/3}, x^{1/5}, \ldots$

$\sqrt[3]{x}, \sqrt[5]{x}, \ldots$



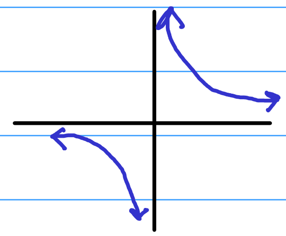
Domain: $\mathbb{R}$

Range: $\mathbb{R}$

5. n is negative and odd:

$x^{-1}, x^{-3}, \ldots$

$\frac{1}{x}, \frac{1}{x^3}, \ldots$



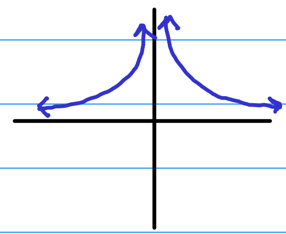
Domain: $\{x : x \neq 0, x \in \mathbb{R}\}$

Range: $\{y : y \neq 0, y \in \mathbb{R}\}$

6. n is negative and even:

$x^{-2}, x^{-4}, \ldots$

$\frac{1}{x^2}, \frac{1}{x^4}, \ldots$



Domain: $\{x : x \neq 0, x \in \mathbb{R}\}$

Range: $(0, \infty)$

Polynomials: Sums of multiples of terms with powers of x that are non-negative integers.

Degree: The highest power of x.

Leading coefficient: The number multiplied by the term with the highest power of x.

Example: $y = -2x^4 + x^3 + 2x^2 - x + 7$ $\quad$ Deg = 4, LC = −2

Fact: If the degree is odd, then:

(a) If the LC is positive, then the left side goes all the way down and the right side goes all the way up (to negative and positive infinity respectively).

(b) If the LC is negative, the opposite happens.

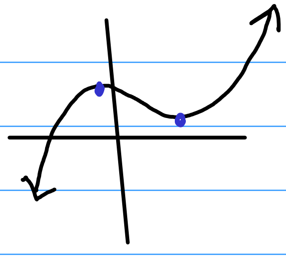
Fact: The maximum number of turning points is one less than the degree.

Fact: If the degree is even, then:

(a) If the LC is positive, then BOTH ends go all the way up.

(b) If the LC is negative, then BOTH ends go all the way down.

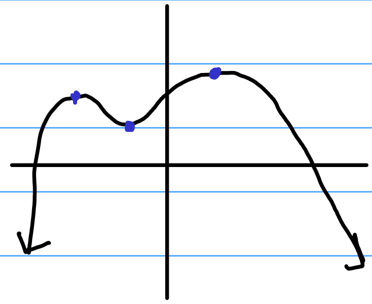
eg



Likely deg = 3 (min)

LC is positive.

eg



Min deg = 4
LC → neg.

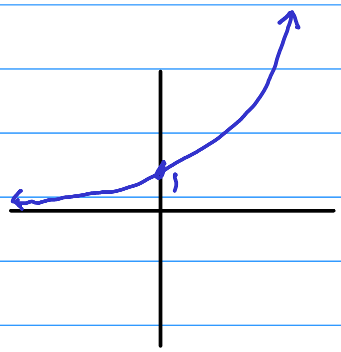
Domain: $\mathbb{R}$

Range: : Deg is odd → $\mathbb{R}$

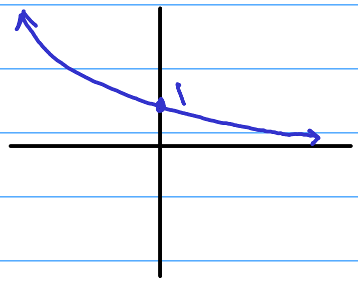
Deg is even → Depends on the formula.

Exponential: $y = a^x$

$a > 1$ eg $y = 2^x$



$0 < a < 1$ eg $y = \left(\frac{1}{3}\right)^x$

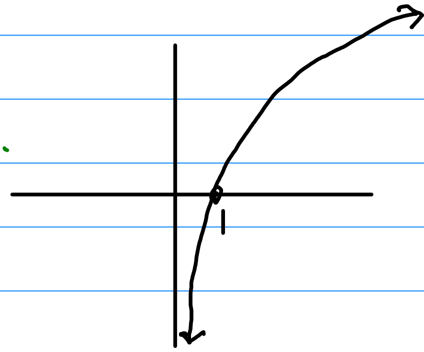


$D = \mathbb{R}$, $R = (0, \infty)$

Logarithmic: $y = \log_a(x)$ (These are the inverse of exponential functions)

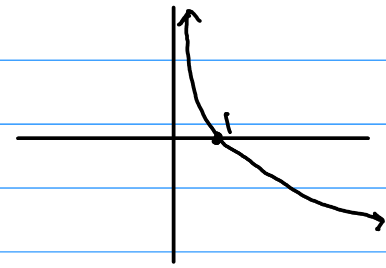
$a > 1$

$\log_3(x)$



$0 < a < 1$:

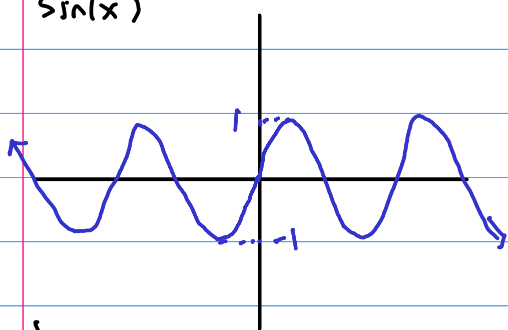
$\log_{\frac{1}{2}}(x)$



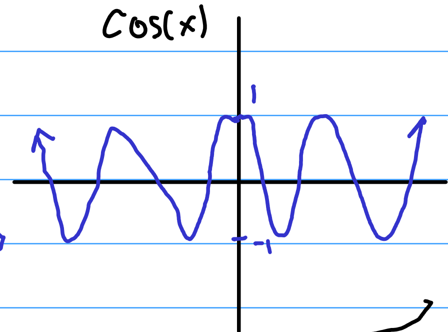
$D = (0, \infty)$, $R = (-\infty, \infty)$

Trigonometric

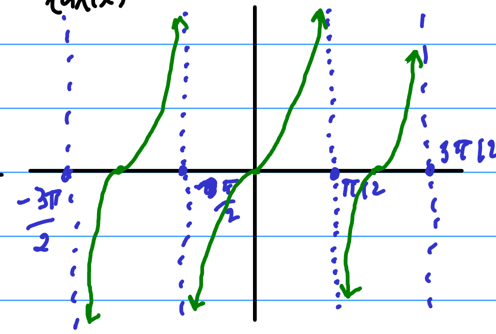
$\sin(x)$



$\cos(x)$



$\tan(x)$



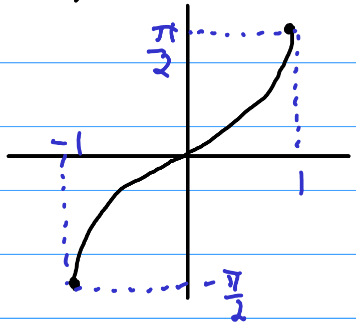
$D = \mathbb{R}$, $R = [-1, 1]$

$D = \{x : x \neq (\text{odd } \#)(\frac{\pi}{2}), x \in \mathbb{R}\}$

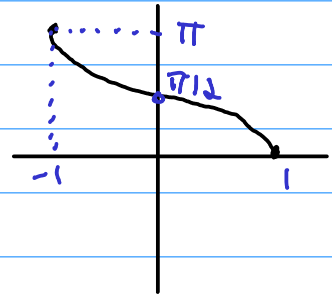
$R = (-\infty, \infty)$

Inverse Trigonometric Functions

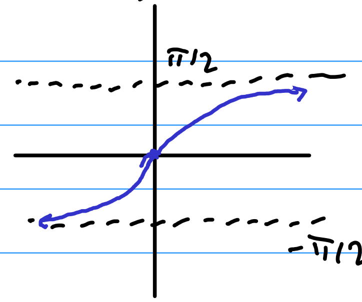
$\arcsin(x)$



$\arccos(x)$



$\arctan(x)$



$D = [-1, 1]$

$R = [-\pi/2, \pi/2]$

$D = [-1, 1]$

$R = [0, \pi]$

$D = (-\infty, \infty)$

$R = (-\pi/2, \pi/2)$

eg Find D. of $y = \sqrt[4]{\underbrace{x-17}_{\text{inside} \geq 0}}$ $\Rightarrow x - 17 \geq 0$

$\boxed{x \geq 17}$

$\Rightarrow D = [17, \infty)$

eg Find D of $\dfrac{\ln(x+3)}{\sqrt{1-x}}$

$\ln(\underbrace{x+3}_{>0})$: $x+3>0 \Rightarrow \boxed{x > -3.}$

$\sqrt{\underbrace{1-x}_{1-x>0}}$: $1-x>0 \Rightarrow 1>x$ or $\boxed{x<1}$

$\Rightarrow D = \{x: -3 < x < 1, x \in \mathbb{R}\}$
or $(-3, 1)$