Logarithms

Recall: Addition and subtraction are inverses (one undoes the other)

Multiplication and division are inverses

Powers and reciprocals of powers are inverses (eg, square and square root)

Now: The inverse of a VARIABLE exponent is a LOGARITHM!

See our more complete coverage of logarithms on our other logarithms video!

$\log_a(x)$ → Produces the power of 'a' required to obtain x.
ie, it answers the Q: $a^{?} = x$

eg $\log_2(8)$ → What power of 2 produces 8?
re $2^{?} = 8$ ⇒ ans = 3. $(2^3 = 8)$
⇒ $\log_2(8) = 3$.

eg $\log_3(21)$ $3^{?} = 21$ ~~$3^1 = 3$~~, ② ~~$3^2 = 9$~~, ③ ~~$3^3 = 27$~~
Somewhere b/w 2 & 3.

Laws

① $\log_a(a^x) = x$, $a^{\log_a(x)} = x$

② $\log_a(xy) = \log_a(x) + \log_a(y)$

③ $\log_a(x/y) = \log_a(x) - \log_a(y)$

④ $\log_a(x^p) = p \cdot \log_a(x)$

⑤ $\log_a(a) = 1$, $\log_a(1) = 0$

eg Solve $3^x = 12$. Take log (any base) of both sides.

$\log_3(3^x) = \log_3(12)$

① $x = \log_3(12)$

OR

④ $\ln(3^x) = \ln(12)$

$x \cdot \ln(3) = \ln(12)$ → $x = \dfrac{\ln(12)}{\ln(3)}$

eg $\log_2(x) = 4$ Raise 2 to power of both sides

$2^{\log_2(x)} = 2^4$

① $\boxed{x = 16}$

eg $\ln(2x) = 3$ $\ln \rightarrow \log_e$

$e^{\ln(2x)} = e^3$

$2x = e^3$

$\boxed{x = \frac{e^3}{2}}$