Polynomial Long Division

Main rule: Highest powers matter most!

$\frac{x^3}{x}$ $\quad$ $-x^2/x$ $\quad$ $\frac{2x}{x}$

eg $\frac{x^3+x}{x+1}$

$$
\begin{array}{r|l}
 & x^2 - x + 2 \\
\hline
x+1 & x^3 + x \\
 & -(x^3 + x^2) \\
\hline
 & -x^2 + x \\
 & -(-x^2 - x) \\
\hline
 & 2x \\
 & -(2x+2) \\
\hline
 & -2
\end{array}
$$

Stop: When remainder's highest power of x is less than highest power of x on the left side

$$\Rightarrow \frac{x^3+x}{x+1} = \underbrace{x^2 - x + 2}_{\text{Quotient}} + \frac{(-2)}{x+1}$$

Remainder

eg $\dfrac{-3x^7+8x^5-x^4+14x^3+2x^2-10x+5}{x^4-2x^2-6}$

$\dfrac{-3x^7}{x^4}$ $\quad$ $\dfrac{2x^5}{x^4}$ $\quad$ $\dfrac{-x^4}{x^4}$

$$\Rightarrow \quad x^4-2x^2-6 \,\big)\!\overline{\;-3x^7+8x^5-x^4+14x^3+2x^2-10x+5\;}$$

$$\begin{array}{rl}
 & -3x^3+2x-1 \\
 & -3x^7+8x^5-x^4+14x^3+2x^2-10x+5 \\
- & (-3x^7+6x^5+18x^3) \\ \hline
 & 2x^5-x^4-4x^3+2x^2-10x+5 \\
- & (2x^5-4x^3-12x) \\ \hline
 & -x^4+2x^2+2x+5 \\
- & (-x^4+2x^2+6) \\ \hline
 & 2x-1
\end{array}$$

Remainder

$$= \underbrace{-3x^3+2x-1}_{\text{Quotient}} + \frac{2x-1}{x^4-2x^2-6}$$