The DEADLY SINS of Algebra!

Here are some common errors, along with their corrected versions. We obviously can't go over every possible error that you can make, but these are the most common. In order to contribute to your instructor's continued health and well being, please stop making these errors! (You know, because we pull our hair out every time we see these mistakes – and that hurts).

This is WRONG!	WRONG Example	This is CORRECT!	CORRECT Example	Notes
$x^m x^n = x^{mn}$	$x^2 x^5 = x^{10}$	$x^m x^n = x^{m+n}$	$x^2 x^5 = x^7$	Multiplying with same base: ADD exponents.
$\frac{x^m}{x^n} = x^{m/n}$	$\frac{x^{12}}{x^3} = x^4$	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^{12}}{x^3} = x^9$	Dividing with same base: SUBTRACT exponents.
$(x^m)^n = x^{(m^n)}$	$(x^3)^2 = x^9$	$(x^m)^n = x^{mn}$	$(x^3)^2 = x^6$	Exponents of exponents: MULTIPLY exponents.
$x^m + x^n = x^{m+n}$	$x^5 + x^2 = x^7$	$x^m + x^n$ cannot be simplified if the exponents are different.	N/A	If the exponents are the same, then you'd get $x^m + x^m = 2x^m$ Example: $x^7 + x^7 = 2x^7$
$(x+y)^n = x^n + y^n$	$(x+y)^2 = x^2 + y^2$ or $\sqrt{x^2+y^2} = x + y$	$(x+y)^n$ must be FOILED (will look different depending on the exponent).	$(x+y)^2 = x^2 + 2xy + y^2$	$\sqrt{x^2+y^2}$ cannot be simplified.
$a(b^x) = (ab)^x$	$3(8^x) = 24^x$	$(ab)^x = a^x b^x$	$(3 \cdot 8)^x = 3^x 8^x$	Exponents distribute in multiplication (and division) as long as it is outside the brackets of **both** numbers.
$(-a)^2 = -a^2$	$(-4)^2 = -16$	$(-a)^2 = a^2$	$(-4)^2 = 16$ while $-4^2 = -16$	If the negative sign in INSIDE the squaring brackets (or any EVEN power), it disappears. If the power is ODD, then the negative always remains: $(-4)^3 = -64$
$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$	$\frac{1}{3+5} = \frac{1}{3} + \frac{1}{5}$	$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$	$\frac{3+5}{2} = \frac{3}{2} + \frac{5}{2}$	You can split numerators, but NOT denominators!

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$\frac{\cancel{a}}{\cancel{a}b+c} = \frac{1}{b+c}$	$\frac{\cancel{4}}{\cancel{4}+5} = \frac{1}{1+5} = \frac{1}{6}$	$\frac{\cancel{a}}{\cancel{a}(b+c)} = \frac{1}{b+c}$	$\frac{\cancel{4}}{\cancel{4}(7+3)} = \frac{1}{7+3} = \frac{1}{10}$	The factor you're cancelling must be multiplied on the ENTIRE numerator and the ENTIRE denominator.
$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$	$\frac{3}{2} + \frac{5}{9} = \frac{3+5}{2+9} = \frac{8}{11}$	$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{3}{2} + \frac{5}{9} = \frac{3(9)+2(5)}{2(9)} = \frac{37}{18}$	This, of course, is finding a COMMON DENOMINATOR. The same applies to subtraction of fractions.
$\ln(a+b) = \ln(a) + \ln(b)$	$\ln(5+8) = \ln(5) + \ln(8)$	$\ln(a+b)$ cannot be simplified.	N/A	Same applies to subtraction, multiplication, and division (but see below). Also, we're using $\ln$ here, but this applies to any base of $\log$.
$\ln(ab) = \ln(a)\ln(b)$	$\ln(2\cdot 5) = \ln(2)\ln(5)$	$\ln(ab) = \ln(a) + \ln(b)$ Provided a and b are BOTH positive.	$\ln(2\cdot 5) = \ln(2) + \ln(5)$	MULTIPLICATION inside the brackets means splitting with ADDITION.
$\ln(a/b) = \ln(a)/\ln(b)$	$\ln(2/5) = \ln(2)/\ln(5)$	$\ln(a/b) = \ln(a) - \ln(b)$ Provided a and b are BOTH positive.	$\ln(2/5) = \ln(2) - \ln(5)$	DIVISION inside the brackets means splitting with SUBTRACTION.
$\ln(ab^t) = t\ln(ab)$	$\ln(5\cdot 7^t) = t\ln(35)$	$\ln(ab)^t = t\ln(ab)$ In general: $\ln(x^t) = t\ln(x)$	$\ln(5\cdot 7)^t = t\ln(35)$	In $\ln(ab^t)$, the t applies to the b only (and not the a), thus you cannot bring it down.
$\frac{\ln(a)}{\ln(b)} = \frac{a}{b}$	$\frac{\ln(4)}{\ln(9)} = \frac{4}{9}$	$\frac{\ln(a)}{\ln(b)}$ cannot be simplified	N/A	You CANNOT cancel $\ln$! It is NOT a number – it is a FUNCTION. The same applies to most other functions (ex: trig functions).
$f(g(x)) = f(x)g(x)$	If $f(x) = x^2$ and $g(x) = 5x$ then $f(g(x)) = x^2 \cdot 5x$ $= 5x^3$	$f(g(x))$ looks different depending on what the functions are, but do NOT multiply them!	If $f(x) = x^2$ and $g(x) = 5x$ then $f(g(x)) = f(5x) = (5x)^2$ $= 25x^2$	$f(g(x))$ is composition, NOT multiplication. You insert the inner function INTO the outer function.
Inverses: $f^{-1}(x) = \frac{1}{f(x)}$	The inverse of $f(x) = 4x+2$ is $f^{-1}(x) = \frac{1}{4x+2}$	Solve for x, and then interchange the names of y and x. The result is the inverse.	The inverse of $f(x) = 4x+2$ is $f^{-1}(x) = \frac{x-2}{4}$ (see method from previous cell)	$f^{-1}(x)$ is the INVERSE, NOT the reciprocal! I realize it's bad notation, because an exponent of -1 usually does mean "reciprocal", but NOT when it comes to functions!

This is WRONG!	WRONG Example	This is CORRECT!	CORRECT Example	Notes
$\arcsin(x) = \frac{1}{\sin(x)}$	$\arcsin(1) = \frac{1}{\sin(1)} = \frac{1}{0.8414} = 1.1884$	$\arcsin(x)$ produces the ANGLE which, when you press "sin" on your calculator, produces x AS A RESULT!	$\arcsin(1) = \pi/2$ because the ANGLE which produces 1 as a result, when you press "sin" on your calculator, is $\pi/2$ (there are others, but it equals $\pi/2$ ONLY. See your class notes for the reason).	$\arcsin(x)$ is the INVERSE of $\sin(x)$, NOT its reciprocal! The same applies to all other inverse trig functions!
If $\sin(x) = y$ then $x = \frac{y}{\sin}$	If $\sin(x) = 0.5$ then $x = \frac{0.5}{\sin}$	If $\sin(x) = y$ then $x = \arcsin(y)$	If $\sin(x) = 0.5$ then $x = \arcsin(0.5) = \pi/6$	"sin" (and any other function, including trig and logarithmic) need a NUMBER associated with it. Simply saying "sin" makes no sense – you need to say "sin of ... (number)"
Dividing by VARIABLES can LOSE solutions to an equation!	$x^3 = x \Rightarrow x^2 = 1$ (dividing by x) $\Rightarrow x = \pm 1$	N/A	$x^3 = x \Rightarrow x^3 - x = 0$ $\Rightarrow x(x^2 - 1) = 0$ $\Rightarrow x = 0,\ x = \pm 1$	Dividing by NUMBERS (constants) is always fine – you will never lose solutions by doing this.
If $ab = c$, then $a = c$ or $b = c$	If $x(x-1) = 2$ then $x = 2$ or $x - 1 = 2$, so $x = 3$	If $ab = 0$, then $a = 0$ or $b = 0$	If $x(x-1) = 0$ then $x = 0$ or $x - 1 = 0$, so $x = 1$	This type of thing only works with ZERO on the right side, and is most common when solving quadratic equations (but can occur in many other places).
Not subtracting properly (insufficient use of brackets).	If $f(x) = x^2 + 2$ and $g(x) = 5x - 6$ then $f(x) - g(x) = x^2 + 2 - 5x - 6$ $= x^2 - 5x - 4$	When subtracting, put BRACKETS around the right side!	If $f(x) = x^2 + 2$ and $g(x) = 5x - 6$ $f(x) - g(x)$ $= x^2 + 2 - (5x - 6)$ $= x^2 + 2 - 5x + 6$ $= x^2 - 5x + 8$	Bracketing also matters when MULTIPLYING and DIVIDING (in order to multiply / divide EVERYTHING together, and not just one thing).